The Term Structure of Expected Inflation Rates

by

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0 INTRODUCTION

• ***Goal***: estimating the term structure of expected inflation rates of investors of riskless non-indexed bonds.

• Earlier attempts:

☞ FRANKEL [1982]: macroeconomic framework, nominal interest rate = real interest rate + expected inflation rate (Irving FISHER's hypothesis).

☞ Stanley FISCHER [1975]: theoretical work on indexed bonds:
nominal interest rate = real interest rate + expected inflation rate – ***interest premium***.
The interest premium may have either sign.

☞ Financial literature confirmed this result within quite different frameworks. To name a few: FAMA AND FARBER [1979], COX, INGERSOLL AND ROSS [1981, 1985, henceforth

CIR], LUCAS [1982], BENNINGA AND PROTOPAPADAKIS [1985], BREEDEN [1986].

☞ Empirical literature neglected the interest premium so far. Exceptions:
EVANS [1998] estimates the time-varying interest premium, but not the expected inflation rate (exogeneous).

☞ REMOLONA, WICKENS AND GONG [1998] estimate both the interest premium and the expected inflation rate (time series, selected nominal and index-linked bonds).

• We estimate the term structures of expected inflation rates and interest premia by means of the extended CIR model at a given moment in time. We proceed in ***three steps***:

1. We use a non-linear optimization to determine the ***nominal instantaneous forward interest rates*** from observed prices of coupon-bearing, non-indexed government

bonds (Delabaen and Lorimier, 1992). This methodology is superior to the known methods because

- ✍ it can explain ***any*** term structure of interest rates.
- ✍ the nominal spot interest rates are determined by numerical ***integration*** rather than differentiation (higher numerical accuracy).

2. We fit the extended CIR model to the nominal term structure obtained in the first step by means of a non-linear regression subject to certain constraints. From the estimated model parameters, the term structure of ***real*** spot interest rates is calculated.
3. The term structures of ***expected inflation rates*** and ***interest premia*** are calculated from the estimated model parameters obtain in the second step.

• Result of ***long-term*** inflation forecast by means of yield curve.

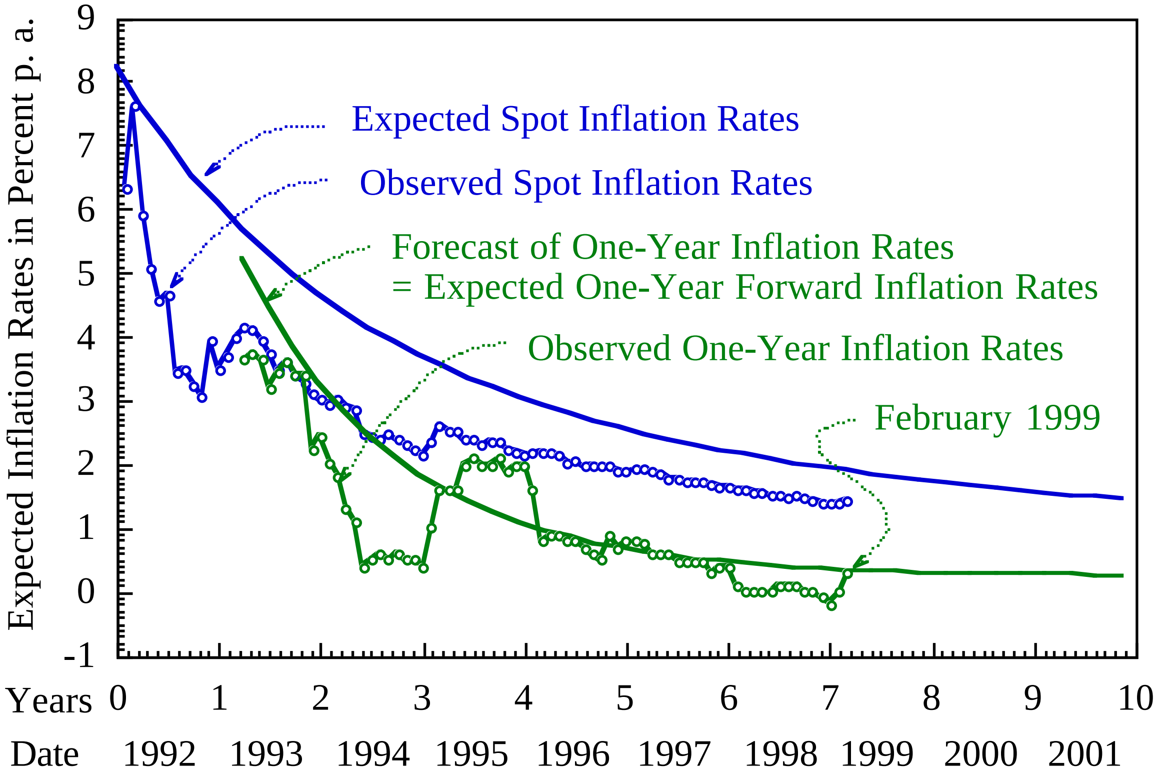

Fig. 6: Inflation Forecast on 30 Dec. 1991

✍ Correct trend.

1 PRELIMINARIES

- ***Spot interest rate***

$$P_\nu(t, T) = \exp\left(-R_{\nu,c}(t, T)\left[T - t\right]\right) \qquad (1\text{-}1)$$

$$R_{\nu,c}(t, T) = -\frac{\ln\left(P_\nu(t, T)\right)}{T - t} \qquad (1\text{-}2)$$

P	Price of a pure discount bond
t	Settlement date
T	Maturity date
R	Spot interest rate
ν	$= n$ (nominal), r (real)
c	Continuous compounding

- ***Instantaneous spot interest rate***

$$r_{\nu,c}(t) \equiv R_{\nu,c}(t, t) = \lim_{t \leftarrow T}\left\{-\frac{\ln\left(P_\nu(t, T)\right)}{T - t}\right\} \qquad (1\text{-}3)$$

r Instantaneous spot interest rate

- ***(τ – T)-year forward interest rate***

$$\mathcal{P}_\nu(t, T, \tau) = \exp\left(-F_{\nu,c}(t, T, \tau)\left[\tau - T\right]\right) \qquad (1\text{-}4)$$

$$F_{\nu,c}(t, T, \tau) = -\frac{\ln(\mathcal{P}_{\nu}(t, T, \tau))}{\tau - T} \tag{1-5}$$

$\mathcal{P}$ The forward price of a pure discount bond, fixed at date t and paid at a later date T when the bond will be delivered. The bond matures at date τ ($\tau \geqq T \geqq t$).

F The ($\tau - T$)-year forward interest rate

$$F_{\nu,c}(t, T, \tau) = \frac{R_{\nu,c}(t, \tau)[\tau - t] - R_{\nu,c}(t, T)[T - t]}{\tau - T} \tag{1-6}$$

- ***Instantaneous forward interest rate***

$$\begin{aligned} f_{\nu,c}(t, T) &= F_{\nu,c}(t, T, T) \\ &= \lim_{\tau \downarrow T} F_{\nu,c}(t, T, \tau) \\ &= R_{\nu,c}(t, T) + \frac{\partial R_{\nu,c}(t, T)}{\partial T}[T - t] \end{aligned} \tag{1-7}$$

f The instantaneous forward interest rate

$$\int_{\tau = t}^{\tau = T} f_{\nu,c}(t, \tau)\, d\tau = R_{\nu,c}(t, T)[T - t] \tag{1-8}$$

$$
\begin{aligned}
P_\nu(t, T) &= \exp\left(-R_{\nu,c}(t, T)\,[T - t]\right) \\
&= \exp\left(-\int_{\tau = t}^{\tau = T} f_{\nu,c}(t, \tau)\, d\tau\right)
\end{aligned}
\qquad (1\text{-}9)
$$

$$
f_{\nu,c}(t, T) = -\frac{\partial \ln\left(P_\nu(t, T)\right)}{\partial T} = -\frac{\dfrac{\partial P_\nu(t, T)}{\partial T}}{P_\nu(t, T)} \qquad (1\text{-}10)
$$

2 Term Structure of Nominal Interest Rates

• ***Task***: calculate the nominal spot interest rates from a sample of observed prices of coupon-bearing bonds.

• ***Known methods***:

☞ Bootstrap method (most popular).

☞ Regression of bond prices on discount factors (Carleton and Cooper [1976]).

☞ Spline methods (McCulloch [1971, 1975], Vasicek and Fong [1982]).

☞ Assumed functional form (Nelson and Siegel [1987], Svensson [1995]).

✍ All these methods have many drawbacks as mentioned in Shea [1984, 1985].

• ***Goal***: in order to obtain a forward rate curve as smooth as possible, minimize the sum of squared differences in instantaneous forward interest rates subject to the constraint that the theoretical bond prices do not deviate from the observed bond prices by more than a given tolerance error (DELABAEN AND LORIMIER [1992], LORIMIER [1995]).

- ***Outline of the Delbaen-Lorimier method***

☞ Suppose we observe L prices of coupon-bearing bonds $B_{obs}(t, T \mid N, c)$, $\ell = 1, 2, \ldots, L$, at date t.

N Face value or redemption value of bond

c Vector of coupons

☞ Ascending order of maturity dates $T_1 < T_2 < \ldots < T_L$.

☞ Let time be divided into equal time steps of length Δt. Determine the number of time steps of length Δt, denoted as H, according to

$$H \equiv \mathcal{B}\left(\frac{T_L - t}{\Delta t}\right) \tag{2-1}$$

☞ We wish to find the instantaneous forward rates at the end points of these time steps. Since $f(t, t) = r(t)$, there are H forward rates to be optimized. For example, with $(T_L - t)$ = 30 years and Δt = 90 days, H = 120.

☞ Theoretical cash price of coupon-bearing bond:

$$B_n\left(t, T_\ell \;\middle|\; N_\ell, c_\ell, q_\ell\right)$$
$$= \sum_{k=0}^{k=\kappa_\ell} c_{\ell,k}\, P_n\left(t, t + \rho_\ell + \frac{k}{q_\ell}\right) + N_\ell\, P_n\left(t, T_\ell\right)$$
$$= \sum_{k=0}^{k=\kappa_\ell} c_{\ell,k} \exp\left(- R_{n,c}\left(t, t + \rho_\ell + \frac{k}{q_\ell}\right)\left[\rho_\ell + \frac{k}{q_\ell}\right]\right)$$
$$+ N_\ell \exp\left(- R_{n,c}\left(t, T_\ell\right)\left[T_\ell - t\right]\right)$$

where

$$\kappa_\ell = \mathcal{B}\left(q_\ell\left[T_\ell - t\right]\right), \quad \rho_\ell = T_\ell - t - \frac{\kappa_\ell}{q_\ell}, \qquad (2\text{-}2)$$

$\ell = 1, 2, \ldots, L$.

B_n	Theoretical price of a nominal coupon-bearing bond
q	Coupon periodicity
κ	Number of coupon dates
ρ	Fraction of the first coupon period

☞ Replace the spot interest rates by the instantaneous forward interest rates:

$$R_{n,c}\left(t, t+\rho_\ell+\frac{k}{q_\ell}\right)\left[\rho_\ell+\frac{k}{q_\ell}\right]$$

$$= \int_{\tau=t}^{\tau=t+\rho_\ell+\frac{k}{q_\ell}} f_{n,c}(t,\tau)\,d\tau, \qquad (2\text{-}3)$$

$$k = 0, \ldots, \kappa_\ell; \quad \ell = 1, \ldots, L.$$

☞ Approximate the integral by the mean value of the upper and lower step function:

$$\int_t^{t+\rho_\ell+\frac{k}{q_\ell}} f_{n,c}(t,\tau)\,d\tau =$$

$$\frac{1}{2}\Delta t \sum_{j=0}^{j=\kappa_{\ell,k}-1} \left[\varphi_j + \varphi_{j+1}\right] + \left[\varphi_{\kappa_{\ell,k}} + \frac{\Delta\varphi_{\kappa_{\ell,k}+1}\,\rho_{\ell,k}}{2\,\Delta t}\right]\rho_{\ell,k}$$

where $\ell = 1, 2, \ldots, L; \quad k = 0, 1, \ldots, \kappa_\ell;$

$$\kappa_{\ell,k} = \mathcal{B}\left(\frac{\rho_\ell + \frac{k}{q_\ell}}{\Delta t}\right), \quad 0 \leqq \kappa_{\ell,k} \leqq H;$$

$$\rho_{\ell,k} = \rho_\ell + \frac{k}{q_\ell} - \kappa_{\ell,k}\,\Delta t \qquad (2\text{-}4)$$

$$\varphi_j = f_{n,c}(t, t + j\,\Delta t)\,,$$

$$(j = 0, 1, \ldots, \kappa_{\ell,k}, \ldots, H); \quad \varphi_{H+1} \equiv \varphi_H.$$

$\kappa\ ^k$ Number of instantaneous forward rates to be considered for the kth coupon date of the ℓth coupon-bearing bond

$\rho\ ^k$ Fraction of the last time step for the kth coupon date of the ℓth coupon-bearing bond

☞ Minimize the squared differences in instantaneous forward interest rates subject to the condition that the relative pricing errors fall into a given tolerance range:

$$\mathcal{F}(\varphi_1, \ldots, \varphi_H) = \min_{\{\varphi_1,\, \varphi_2,\, \ldots,\, \varphi_H\}} \left\{ \sum_{j=1}^{j=H} \left(\Delta\varphi_j\right)^2 \right\}$$

where $\Delta\varphi_j \equiv \varphi_j - \varphi_{j-1}$, φ_0 given,

subject to

$$-\epsilon_\ell \leqq \left(\frac{B_n\left(t, T_\ell \,\middle|\, N_\ell, c_\ell, q_\ell\right)}{B_{\text{obs}}\left(t, T_\ell \,\middle|\, N_\ell, c_\ell, q_\ell\right)} - 1 \right) 100 \leqq \epsilon_\ell \qquad \text{(2-5)}$$

$\ell = 1, 2, \ldots, L.$

ϵ Pricing error tolerances, ($\ell = 1, 2, \ldots, L$).

- ***Results of the DL methodology***

☞ Many expirements applied to the theoretical term structure proposed by VASICEK [1977].

✍ The Delbaen-Lorimier methodology performs much better than the bootstrap method when the term structure is sufficiently bent.

✍ The Delbaen-Lorimier methodology is able to extract ***any*** term structure from a sample of bond prices.

☞ ***Example***: wave-like term structure.

• Instantaneous forward interest rate:

$$f_{n,c}(t, T) = a + b\,(T - t) + \frac{1}{100}\sin\left(c + d\,(T - t)\right)$$

where

$$a = 0.01, \quad b = 0.002667, \qquad (2\text{-}6)$$

$$c = \frac{2\,\pi}{1000}, \quad d = \frac{4\,\pi}{15}.$$

• Spot interest rate:

$$R_{n,c}(t, T) = a + \frac{1}{2}\,b\,(T - t)$$

$$- \frac{2}{d\,(T - t)\,100} \qquad (2\text{-}7)$$

$$\times \sin\left(c + \frac{d\,(T - t)}{2}\right)\sin\left(- \frac{d\,(T - t)}{2}\right)$$

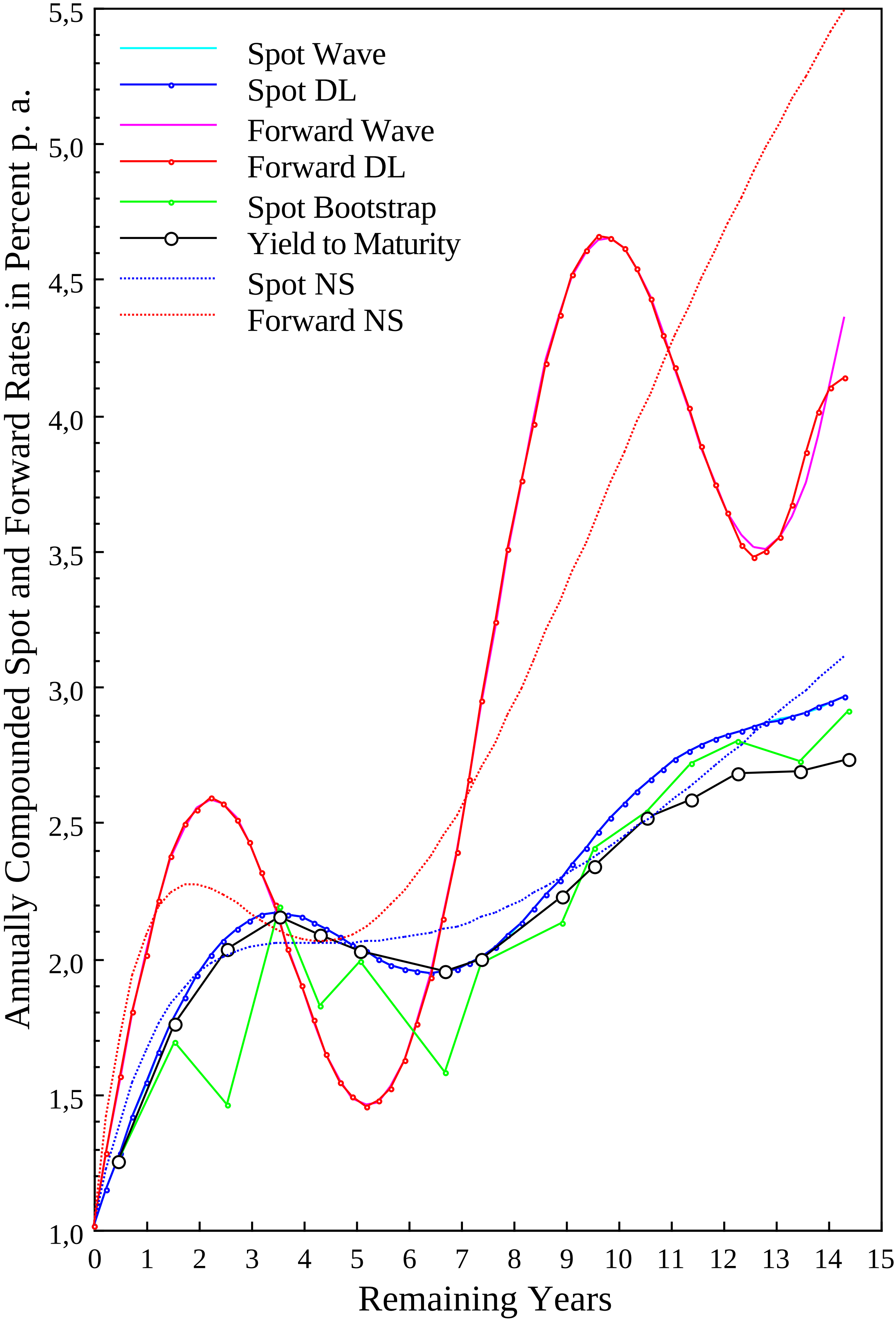

Figure 1: Wave Term Structure

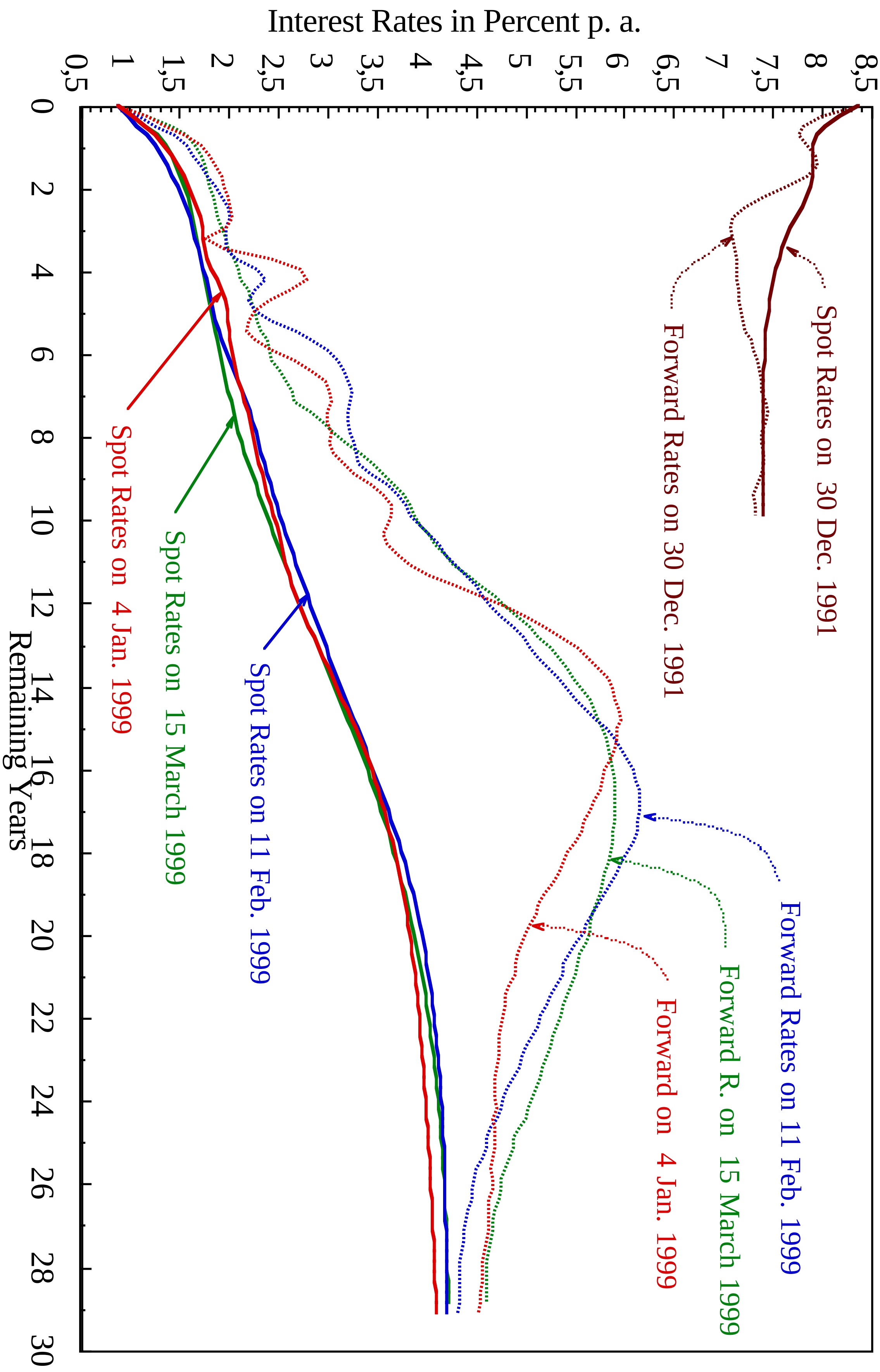

Figure 2: Term Structure of Nominal Interest Rates

- ***Possible Explanations of the steep yield curve***

☞ Domestic Inflation? Unlikely.

☞ Imported Inflation? May be.

☞ Adjustment to higher interest rates in the EU?

3 Term Structure of Real Interest Rates

• If there are ***no*** indexed bonds traded in financial markets, then we must rely on a model which is able to explain nominal and real interest rates simultaneously. Two candidate models: CIR [1985] and Bakshi and Chen [1996].

• ***Outline of the CIR model 2***

☞ Real instantaneous spot interest rate:

$$dr_{r,c}(t) = \varkappa\left[\theta - r_{r,c}(t)\right]dt + \sigma\sqrt{r_{r,c}(t)}\,dz_1(t)\,, \qquad 0 \leqq \varkappa,\ \theta,\ \sigma < \infty \tag{3-1}$$

r_c^r	Continuously compounded real instantaneous spot interest rate
$\varkappa$	Speed of adjustment
θ	Long-run equilibrium value
σ	Constant volatility parameter
z_1	Gauss-Wiener process

☞ Price of a ***real*** pure discount bond:

$P_r(t, T) = A(t, T)^{\psi} \exp\left(- B(t, T)\, r_{r,c}(t)\right)$, where

$$A(t, T) = \frac{2\,\gamma \exp\left(\frac{[\varkappa + \lambda + \gamma]\,[T - t]}{2}\right)}{[\varkappa + \lambda + \gamma]\,[\exp(\gamma\,[T - t]) - 1] + 2\,\gamma}$$

$$B(t, T) = \frac{2\,[\exp(\gamma\,[T - t]) - 1]}{[\varkappa + \lambda + \gamma]\,[\exp(\gamma\,[T - t]) - 1] + 2\,\gamma} \qquad (3\text{-}2)$$

$$\gamma = \sqrt{[\varkappa + \lambda]^2 + 2\,\sigma^2}\,, \quad \psi = \frac{2\,\varkappa\,\theta}{\sigma^2}$$

P_r Price of a real pure discount bond

λ Factor risk premium

☞ Processes for the consumer price level and the inflation rate:

$$\begin{aligned} &\mathrm{d}r_{y,c}(t) = \varkappa_2\,[\theta_2 - r_{y,c}(t)]\,\mathrm{d}t + \sigma_2 \sqrt{r_{y,c}(t)}\,\mathrm{d}z_3(t)\,, \\ &0 \leqq \varkappa_2,\ \theta_2,\ \sigma_2 < \infty \\ &\mathrm{d}p(t) = r_{y,c}(t)\,p(t)\,\mathrm{d}t + \sigma_p\,p(t) \sqrt{r_{y,c}(t)}\,\mathrm{d}z_2(t)\,, \\ &\mathscr{C}\{\mathrm{d}r_{y,c},\ \mathrm{d}p\} = \zeta\,\sigma_2\,\sigma_p\,r_{y,c}\,p, \quad 0 \leqq \sigma_p < 1 \end{aligned} \qquad (3\text{-}3)$$

r_c^y Instantaneous spot inflation rate

$\varkappa_2$	Speed of adjustment
θ_2	Long-run equilibrium value
σ_2	Constant volatility parameter
z_3	Gauss-Wiener process
p	Consumer price level
σ_p	Constant volatility parameter
z_2	Gauss-Wiener process
ζ	Correlation coefficient between z_2 and z_3
$\mathcal{C}$	Covariance operator

☞ Price of a ***nominal*** pure discount bond:

$$P_n(t, T) = P_r(t, T)\, C(t, T)^{\psi_2} \times \exp\left(-D\left(t, T, r_{y,c}(t)\right)\right)$$

$$C(t, T) = \frac{2\,\xi \exp\left(\dfrac{\left[\varkappa_2 + \zeta\,\sigma_2\,\sigma_p + \xi\right]\left[T - t\right]}{2}\right)}{\left[\varkappa_2 + \zeta\,\sigma_2\,\sigma_p + \xi\right]\left[\exp\left(\xi\left[T - t\right]\right) - 1\right] + 2\,\xi}$$

$$D\left(t, T, r_{y,c}(t)\right) = \frac{2\left[\exp\left(\xi\left[T - t\right]\right) - 1\right]\left[1 - \sigma_p^2\right] r_{y,c}(t)}{\left[\varkappa_2 + \zeta\,\sigma_2\,\sigma_p + \xi\right]\left[\exp\left(\xi\left[T - t\right]\right) - 1\right] + 2\,\xi}$$

$$\xi = \sqrt{\left[\varkappa_2 + \zeta\,\sigma_2\,\sigma_p\right]^2 + 2\,\sigma_2^2\left[1 - \sigma_p^2\right]} \qquad (3\text{-}4)$$

$$\psi_2 = \frac{2\,\varkappa_2\,\theta_2}{\sigma_2^2}$$

P_n Price of a nominal pure discount bond

☞ Define 11-by-1 parameter vector

$$\boldsymbol{\beta} = [\varkappa, \theta, \sigma, \lambda, \varkappa_2, \theta_2, \sigma_2, \sigma_p, \zeta, r_c^r(t), r_c^y(t)]'$$

☞ Spot interest rates in the CIR model

$$R_{r,c}\left(t, T \mid \boldsymbol{\beta}\right) = \frac{B(t, T)\, r_{r,c}(t) - \psi \ln\left(A(t, T)\right)}{T - t}$$

$$R_{n,c}\left(t, T \mid \boldsymbol{\beta}\right) = R_{r,c}\left(t, T \mid \boldsymbol{\beta}\right) + \frac{-\psi_2 \ln\left(C(t, T)\right) + D\left(t, T, r_{y,c}(t)\right)}{T - t} \qquad (3\text{-}5)$$

R^r_c Real spot interest rate

R^n_c Nominal spot interest rate

☞ Take the limit as $T \rightarrow t$

$$R_{r,c}\left(t, t \mid \boldsymbol{\beta}\right) = r_{r,c}\left(t\right)$$

$$R_{n,c}\left(t, t \mid \boldsymbol{\beta}\right) = r_{n,c}\left(t\right) = r_{r,c}\left(t\right) + \left[1 - \sigma_p^2\right] r_{y,c}(t) \qquad (3\text{-}6)$$

r^r_c Real instantaneous spot interest rate

r^n_c Nominal instantaneous spot interest rate

☞ Process for the nominal instantaneous spot interest rate

$$\begin{aligned} \mathrm{d}r_{n,c}(t) = {} & \left\{\varkappa \left[\theta - r_{r,c}(t)\right] + \varkappa_2 \left[\theta_2 - r_{y,c}(t)\right]\right\} \mathrm{d}t \\ & + \sigma \sqrt{r_{r,c}(t)}\, \mathrm{d}z_1(t) \qquad (3\text{-}7) \\ & + \left[1 - \sigma_p^2\right] \sigma_2 \sqrt{r_{y,c}(t)}\, \mathrm{d}z_3(t) \end{aligned}$$

☞ Find parameter vector $\boldsymbol{\beta}$ by constrained regression

$$\min_{\{\boldsymbol{\beta}\}} \left\{ \sum_{j=1}^{H} \left[R_{n,c}(t, j \mid \boldsymbol{\beta}) - R_{n,c}(t, j) \right]^2 \right\}$$
$$R_{n,c}(t, j \mid \boldsymbol{\beta}) \equiv R_{n,c}(t, t + j\,\Delta t \mid \boldsymbol{\beta})$$
$$R_{n,c}(t, j) \equiv R_{n,c}(t, t + j\,\Delta t)$$
$$s.\,t. \tag{3-8}$$
$$R_{n,c}(t, t + \boldsymbol{\iota}(k)\,\Delta t \mid \boldsymbol{\beta}) = R_{n,c}(t, t + \boldsymbol{\iota}(k)\,\Delta t)\,,$$
$$k = 1, \ldots, m; \quad m \leqq 11.$$

$\boldsymbol{\iota}$ Indicator vector which locates the interest rates to be constrained

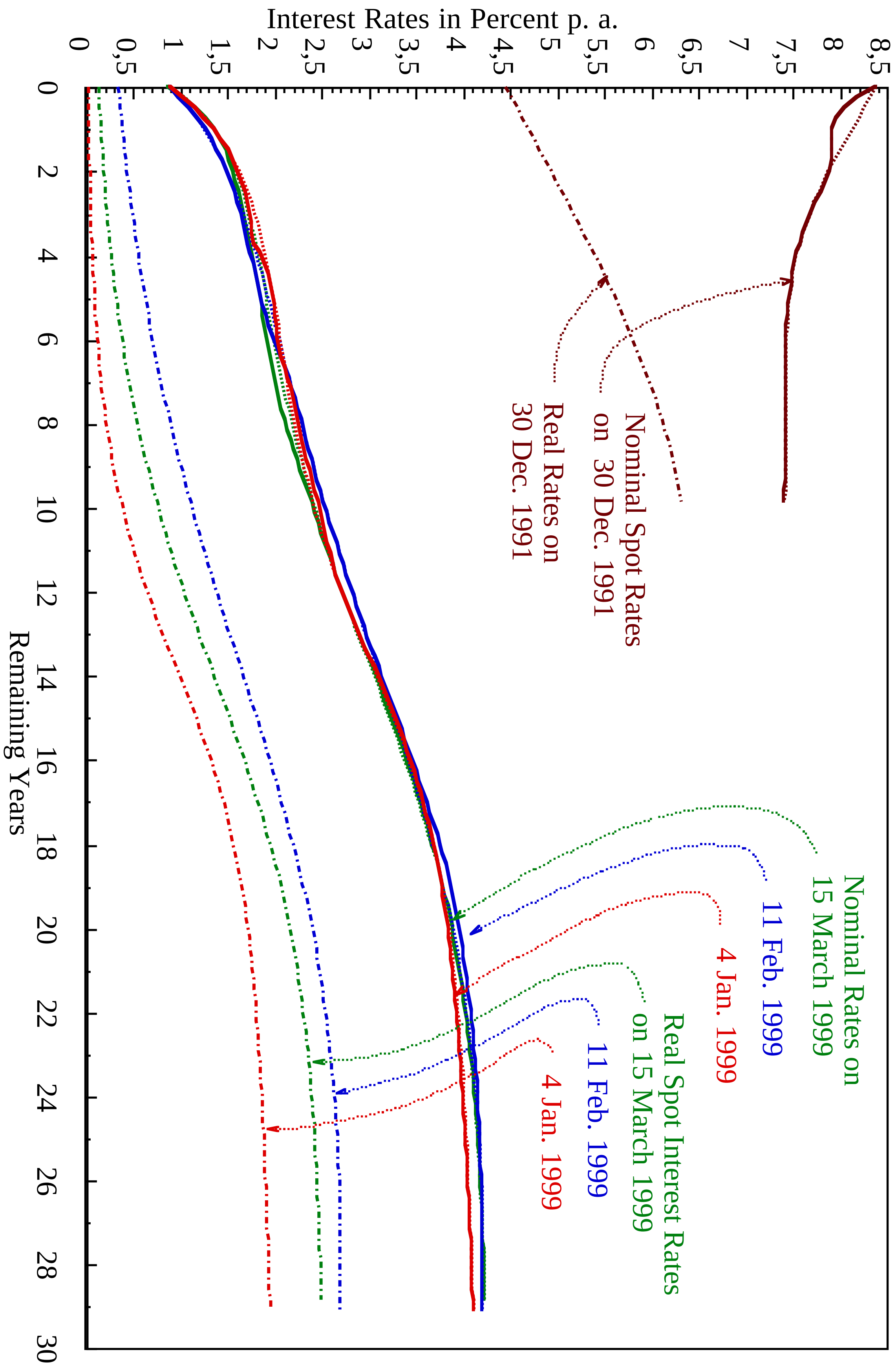

Figure 3: Real Spot Interest Rates

4 Expected Inflation Rates

- Define ***random*** inflation rates correspondingly

$$p(t) = \exp\left(-R_{y,c}(t,T)\,[T-t]\right)\,p(T)$$
$$= \exp\left(-\int_{\tau=t}^{\tau=T} f_{y,c}(t,\tau)\,d\tau\right)\,p(T)\,, \qquad (4\text{-}1)$$
$$t \leqq T.$$

p Consumer price level

R^{y}_{c} Spot inflation rate

f^{y}_{c} Instantaneous forward inflation rate

- ***Expected*** values of inflation rates

$$\mathcal{E}\{R_{y,c}(t,T)\}\,[T-t] = \int_{\tau=t}^{\tau=T} \mathcal{E} f_{y,c}(t,\tau)\,d\tau$$
$$= \mathcal{E}\ln\left(\frac{p(T)}{p(t)}\right) \qquad (4\text{-}2)$$

$\mathcal{E}$ Expectation operator

- Since $\mathcal{E}\ln(p(T)/p(t)) \leqq \ln(\mathcal{E}\{p(T)/p(t)\})$ by Jensen's inequality, relationship (4-1) reads for the expected values as follows:

$$p(t) = \exp\left(-\mathscr{E}\left\{R_{y,c}(t, T)\right\}[T-t]\right) \times \exp\left(\mathscr{E}\ln\left(p(T)\right)\right) = \exp\left(-\int_{\tau=t}^{\tau=T} \mathscr{E} f_{y,c}(t, \tau)\, d\tau\right) \times \exp\left(\mathscr{E}\ln\left(p(T)\right)\right) \quad (4\text{-}3)$$

• The expected instantaneous ***spot*** inflation rate is equal to the expected instantaneous ***forward*** inflation rate:

$$\ln\left(p(t)\right) + \int_{\tau=t}^{\tau=T} f_{y,c}(t, \tau)\, d\tau = \ln\left(p(T)\right) \quad (4\text{-}4)$$

☞ Differentiate with respect to "maturity" date

$$f_{y,c}(t, T)\, dt = d\ln\left(p(T)\right) = \frac{dp(T)}{p(T)} \quad (4\text{-}5)$$

☞ Equ. (4-5) is stochastic equivalence to equ. (1-10). Take expectations:

$$\mathscr{E} f_{y,c}(t, T) = \mathscr{E} r_{y,c}(T), \quad t \leqq T. \quad (4\text{-}6)$$

• Obtain the expected instantaneous ***spot*** inflation rate by recurrence for small time steps:

$$\mathcal{E} r_{y,c}(t + j\,\Delta t) = \left[1 - \varkappa_2\,\Delta t\right] \mathcal{E} r_{y,c}\left(t + [j-1]\,\Delta t\right) + \varkappa_2\,\theta_2\,\Delta t\,, \quad j = 2, 3, \ldots$$

$$\mathcal{E} r_{y,c}(t + \Delta t) = \left[1 - \varkappa_2\,\Delta t\right] r_{y,c}\left(t\right) + \varkappa_2\,\theta_2\,\Delta t\,, \quad j = 1. \tag{4-7}$$

• The ***expected*** instantaneous forward inflation rates define the whole ***term structure*** of ***expected*** inflation rates.

• Finally, the ***interest premium*** becomes

$$R_{n,c}\left(t, T\right) = R_{r,c}\left(t, T\right) + \mathcal{E} R_{y,c}\left(t, T\right) - \eta_c\left(t, T\right) \tag{4-8}$$

η_c Continuously compounded interest premium

• In the CIR framework, the ***interest premium*** consists of two terms, namely,

✍ of the ***variance*** of the consumer price level and

✍ of the so-called ***wealth premium***. The wealth premium depends on the investor's attitude towards risk as well as on the covariance between real wealth and the future inflation, which have either sign.

✍ Hence, the interest premium may be positive ***or*** negative.

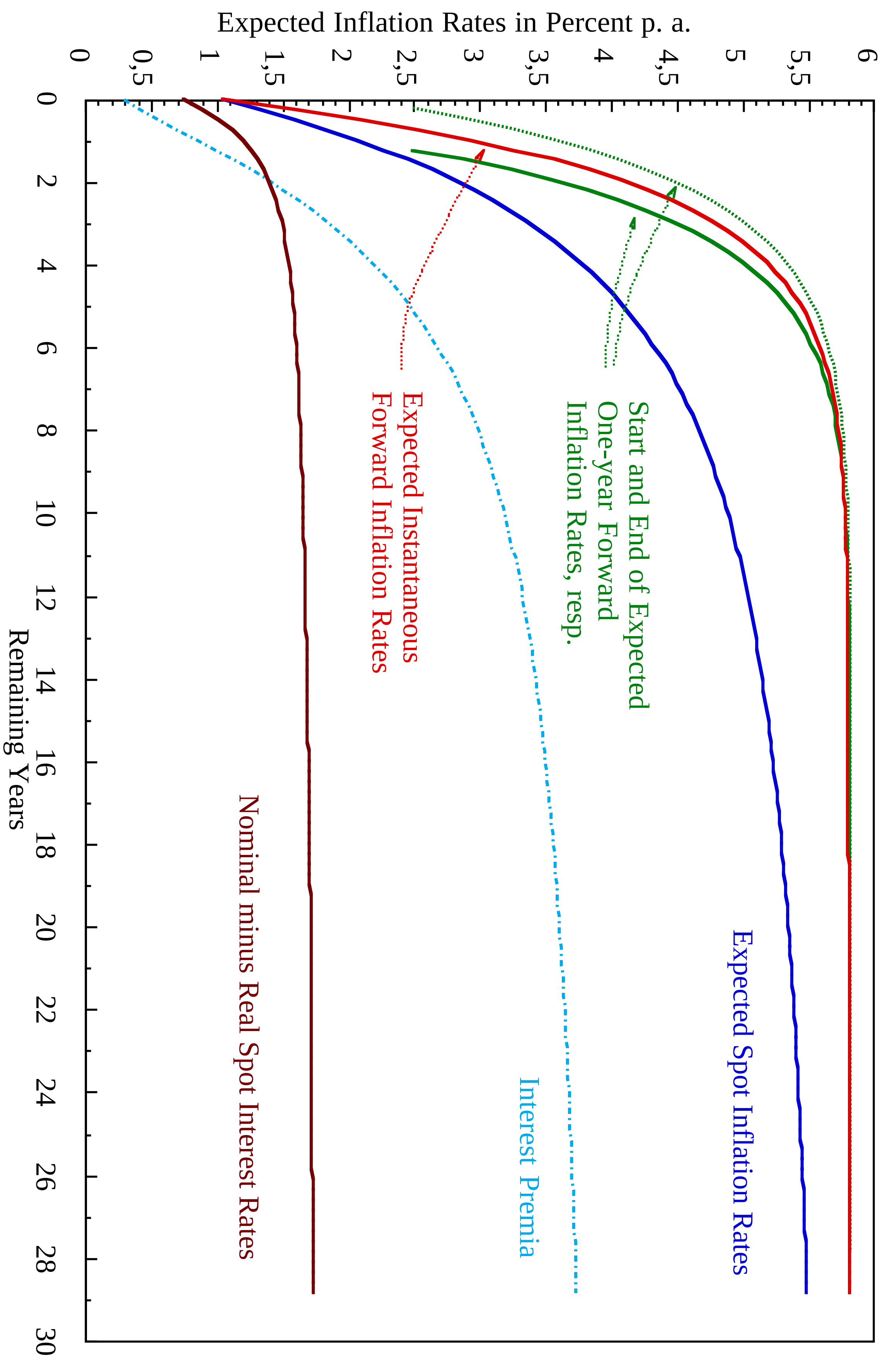

Figure 4: Expected Inflation Rates on 15 March 1999

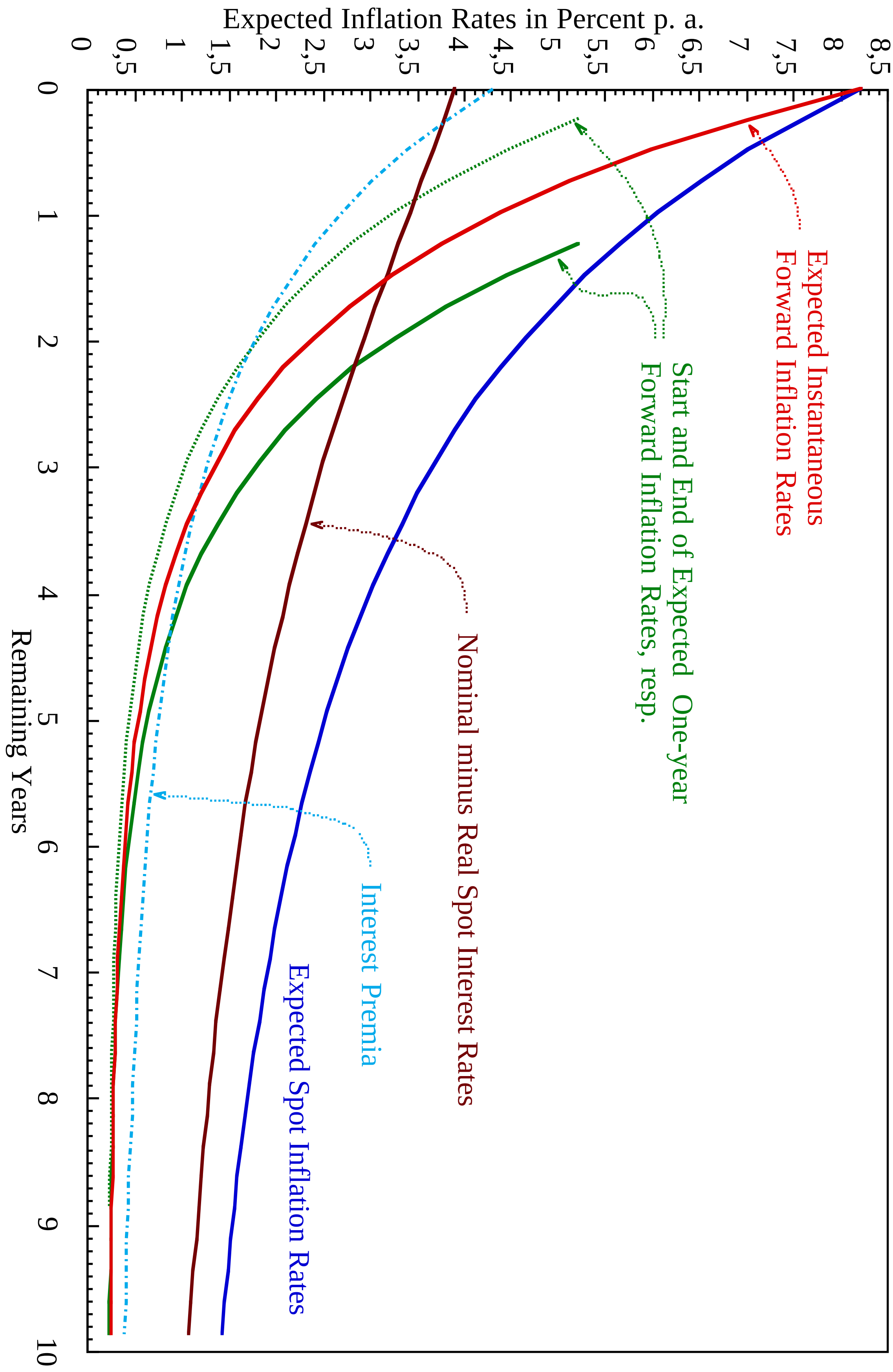

Figure 5: Expected Inflation Rates on 30 December 1999

• Why are the ***present*** inflationary expectations so high? We offer three possible explanations for this phenomenon.

1. Investors of government bonds might expect a high future ***domestic*** inflation.
   - ✍ In view of the present non-existent inflation, and in view of the present slackness of the Swiss economy, this seems rather an unplausible explanation.
2. The investors might expect a high foreign, that is, ***imported*** inflation.
   - ✍ Again, this seems an unplausible explanation because the inflation is presently very low in those European countries which are the main trading partners of Switzerland.
3. The investors might expect the Swiss nominal spot rates to adhere to the higher interest rate level of the European Union (EU), in particular,

because they might expect that Switzerland will join the EU in the near future.

✍ We feel that the third explanation is the most conceivable one for three reasons.

a. The CIR model to evaluate the term structure is a model of a closed economy as well as a model that is not able to explain other effects than the expected inflation rate and the interest premium.

b. A possible Swiss memberhip in the EU was not a political issue at the end of 1991. Hence, the term structure considered for this date should not contain the possible effect that the Swiss interest rates could adhere to the higher interest rate level of the EU. The term structure considered at the end of 1991 forecasts very low future inflation rates of about 0.3%

per annum for the years 1999 to 2001 as shown in figure 6.

c. An inspection of the variance of the consumer price level in the course of future time, shows that this variance calculated for a recent term structure is twice as big as the variance for the term structure considered at the end of 1991. Hence, the investor's uncertainty about future variations of the consumer price level has risen substantially, although the inflation rate fell substantially during the past decade.

5 Conclusions

- In this paper, we infer the term structure of ***expected inflation rates*** from a sample of observed prices of coupon-bearing bonds by means of a three-step procedure.
- In the first step, the nominal instantaneous forward interest rates are optimized.
- In the second step, the extended CIR model is fitted to the optimized nominal spot interest rates. By this procedure, the real spot interested rates are determined.
- Finally, the estimated CIR model parameters allow the calculation of the expected instantaneous forward inflation rates by means of a simple recurrence relationship.
- The expected inflation rates coincide quite well with the observed inflation rate for a sample observed at the end of 1991.

- We conjecture that the present high expected inflation rate obtained from the CIR model is a ***spurious*** inflationary expectation. Rather, the result relates to the difference in interest rates between Switzerland and the EU countries.
- Therefore, we plan to investigate the ***European*** term structure of expected inflation rates.