Classifying Corporate Bonds: A Simple Approach

by

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ABSTRACT

This paper introduces a simple approach to classify corporate bonds. It relies entirely on the quoted price as well as on the quoted bid and ask price of corporate bonds. If current bond prices reflect all the relevant corporate information, then this approach is sufficient for a credit rating which is also up-to-date. We apply the approach outlined in the paper to the group of coupon-bearing bonds issued by banks as well as to other groups of corporate bonds including industrial companies and federal states. Our five-year experience shows that our algorithm works well, even for illiquid markets.

KEYWORDS: Corporate bond, credit rating, term structure of interest rates

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0 Introduction

The credit rating provided by one of the well-known rating agencies relies on the detailed information as regards the financial status of a particular corporation. To quote from Standard & Poor's [2006], "the rating process is not limited to an examination of various financial measures. Proper assessment of credit quality for an industrial company includes a thorough review of business fundamentals, including industry prospects for growth and vulnerability to technological change, labor unrest, or regulatory actions. In the public finance sector, this involves an evaluation of the basic underlying economic strength of the public entity, as well as the effectiveness of the governing process to address problems. In financial institutions, the reputation of the bank or company may have an impact on the future financial performance and the institution's ability to repay its obligations."

It should be noted that the agencies evaluate separately each bond issue of a particular corporation rather than the whole corporation under consideration. This is due to the legal status of each bond issue in the case of a default of a corporation.[1] Apart from the legal status, the financial markets often prefer particular bonds of a corporation over others due to different terms to maturity (or tenors, respectively) and/or coupons. Hence, market prices of bonds may reflect both the creditworthiness of a corporation and the investors' preferences for a particular bond of this corporation.

Due to the considerable cost, the rating procedure is repeated rather infrequently. As a consequence, the rating follows quite often the financial development of a corporation with a considerable time lag. In particular, in a situation of financial stress, the bond of a corporation might be traded at a price which indicates that the corporation has been downgraded by the market well before a rating agency will release the downgrade of the corporation under consideration. This has cast some doubt on the importance of rating agencies in the literature. To quote from Brealy and Myers [2003, p. 685], "firms and governments ... almost certainly exaggerate the influence of rating agencies, which are as much following investor opinion as leading it".

The existing literature on credit rating is primarily concerned with the reaction of corporate bond and stock prices to the announcement of changes of the credit rating. For instance, Hite and Warga [1997] study the bond price reaction to announcements of the credit rating agencies for U. S. corporate bonds. They find that announcements of a downgrade have a much stronger effect on bond prices than upgrades. Price reactions also occur during the preannouncement period, meaning that investors rely on other information as well. Odders-White and Ready [2006] analyze the relationship between credit ratings and measures of equity mar-

1 In the United States, senior secured bonds, senior unsecured bonds, senior subordinated bonds, subordinated bonds and junior subordinated bonds have different priorities in meeting the claims against the assets of a particular corporation. I am indebted to François-Marie Monnet for pointing this out to me.

ket liquidity. They find that common measures of adverse selection, i. e., the risk of market makers of facing traders with better information, are larger when credit ratings are poorer. They also show that future credit rating changes can be predicted by means of current levels of adverse selection measures and the ratio of equity to assets. In their view, the adverse selection measures may be a useful tool for assessing credit risk on a more timely basis. Leake [2003] studies the relationship between credit spreads of UK corporate bonds and the term structure of interest rates. He finds that the credit spreads have been driven by factors other than default risk. In an empirical investigation of the primary Eurobond market, Gabbi and Sironi [2002] find that the most important determinant of the corporate bond price is the credit rating. They also find that the credit rating agencies adopt a “through the business cycle” approach to evaluate the creditworthiness of a corporation in contrast to the bond investors who adopt a forward looking approach. Emery and Cantor [2005] find that the default rates for loans are smaller than those for U. S. non-financial corporate bonds. This result is due to the fact that there exist companies in the coprorate bond market who default on their bonds but avoid to go bancrupt and thus avoid to default on their loans.

The predictability of future credit ratings has been addressed by Chan and Jegadeesh [2005]. They investigate the power of some statistical methods[2] to predict the credit rating of U. S. corporations for a five-year forecasting time horizon. They argue that agency ratings are a poor benchmark because they are slow to reflect new company information. To evaluate the forecasting models considered, the authors construct a so-called “relative rating strength portfolio” which, on the one hand, consists of long bonds whose predicted ratings are better than those given by the rating agencies, and, on the other hand, consists of short bonds whose predicted ratings are worse than those given by the rating agencies. In the authors’ view, the “relative rating strength portfolio” should produce abnormal profits. The authors find some kind of inefficiency in the U. S. corporate bond market which is due to the fact that investors rely too much on the rating agencies which, in turn, are too slow in providing new corporate information.

The question of the economic value of the credit rating agencies has been addressed by Boot, Milbourn and Schmeits [2005] in the framework of a theoretical model. They argue that the credit rating agencies play an economically meaningful role in that they serve as a coordinating mechanism in situation where multiple equilibria can obtain. They consider an implicit contractual framework between a credit rating agency and a corporation through its credit watching procedure. By this, they resolve some empirical contrarieties as regards the reaction of bond and stock market prices to announcements of the credit rating agencies, either upgrades or downgrades.

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[2] The statistical methods considered by Chan and Jegadeesh [2005] are the ordinary least-squares method, the multiple discriminant analysis, the multiple discriminant analysis including cross-validation, the probit model as well as the probit model with stepwise variable selection.

In this paper, we propose a simple approach to classify corporate bonds. This approach relies entirely on the quoted prices of the corporate bonds as well as on their quoted bid and ask prices. Hence, we assume some kind of market efficiency in that the currenct bond prices reflect the relevant information as regards the creditworthiness of a company. Further, this classification of the bonds incorporates both the creditworthiness and the investors' preferences. It may happen that bonds of equal indenture fall into different classes due to investors' preferences. Our approach follows the idea of estimating a yield curve for coupon-bearing bonds described in the appendix. To our knowledge, there exist no other paper which addresses the question of classifying corporate bonds by observed bond prices.

The outline of the paper is as follows. In the next section, we describe the algorithm of classifying corporate bonds. In the second section, we apply the classification method to a sample of bank bonds and then estimate the yield curve for each class. In the last section, we conclude that the classification method worked well for a five-year period. The appendix describes the method used in this paper to estimate the yield curve from a sample of coupon-bearing bonds.

1 Credit Classes

The basic idea is to divide a group of corporate bonds into different classes by means of the bond yield or the yield to maturity of a coupon-bearing bond, respectively. The bid price sets an upper bound for the possible bond yields and the ask price sets a lower bound for the possible bond yields, given the condition that the estimated yield curve of each class should lie within the range of bid and ask prices of the bonds included in a particular class. For each bond, therefore, we get an interval (set) of possible bond yields. Corporate bonds of the same class must have a non-empty intersection set of their bond yield sets, whereas corporate bonds belonging to different classes must have an empty intersection set. In the sequel, we describe the steps of the algorithm.

At the beginning, we exclude those corporate bonds from the sample whose terms to maturity or tenors, respectively, are less than one year because these bonds are traded very infrequently.

The bond yield, denoted as y, is defined as that constant discount rate which equates the present value of the cash flows with the observed bond price, denoted as $B^{\text{obs}}(\tau)$, where τ denotes the tenor of the coupon-bearing bond, that is,

$$B^{\text{obs}}(\tau) = \sum_{k=1}^{K} c_k \exp\left(-y(\tau)\,\tau_k\right), \quad (0 \leqq \tau_k \leqq \tau) \tag{1}$$

where c_k denotes the kth coupon and τ_k the corresponding tenor. Note that the last coupon includes the redemption value by definition. The bond yield assumes that all the cash flows can be discounted by the same yield. It is a zero-coupon rate if, and only if, the term structure of interest rate is flat. It can readily be computed. The first check is to exclude those corporate bonds from the remaining sample whose bond yields, if any, are negative.

Since we require the yield curve of each class to lie within the range of the corresponding bid-ask prices of corporate bonds, denoted as B^{bid} and B^{ask}, we calculate the bid-price based bond yields and the ask-price based bond yields as well.

$$\begin{aligned} B^{\text{bid}}(\tau) &= \sum_{k=1}^{K} c_k \exp\left(-\, y^{\text{bid}}(\tau)\ \tau_k\right) \\ B^{\text{ask}}(\tau) &= \sum_{k=1}^{K} c_k \exp\left(-\, y^{\text{ask}}(\tau)\ \tau_k\right) \end{aligned} \tag{2}$$

where y^{bid} denotes the bid-price based bond yield and y^{ask} denotes the ask-price based bond yield.

Let $y^{*}(\tau_m)$ denote the smoothed bond yield of the government bonds which corresponds with the term to maturity of the *m*th corporate bond.[3] The bond-yield based credit risk premium[4] or the spread of corporate bond yields over government bond yields, respectively, is denoted as π, that is,

$$\begin{aligned} \pi^{\text{bid}}(\tau) &= y^{\text{bid}}(\tau) - y^{*}(\tau) \\ \pi(\tau) &= y(\tau) - y^{*}(\tau) \\ \pi^{\text{ask}}(\tau) &= y^{\text{ask}}(\tau) - y^{*}(\tau) \end{aligned} \tag{3}$$

where π^{bid} denotes the bid-price based credit risk premium and π^{ask} denotes the ask-price based credit risk premium. The second check is to exclude those corporate bonds from the remaining sample whose credit risk premia (π), if any, are negative. By this, we assume that the “sovereign ceiling” rule applies. However, the ask-price based credit risk premium (π^{ask}) may become negative.

The third check excludes those corporate bonds from the remaining sample whose bid-ask premium spreads are outliers. Let $\pi^{\text{bid-ask}}$ denote the spread of the bid-price based credit risk premium over the ask-price based credit risk premium, that is,

$$\pi^{\text{bid-ask}}(\tau) = \pi^{\text{bid}}(\tau) - \pi^{\text{ask}}(\tau) \tag{4}$$

An empirical density distribution of the observed bid-ask premium spreads is depicted in Figure 1a and the corresponding cumulative distribution in Figure 1b for a sample of 382 corporate bonds observed on 22-Nov-2002. A comparison of the empirical distribution with the lognonormal distribution in Figures 1a and 1b shows a sufficient coincidence for our purposes.

3 We use a smoothed cubic spline to interpolate the bond yields obtained from all the government bonds outstanding.

4 Also denoted as the credit spread.

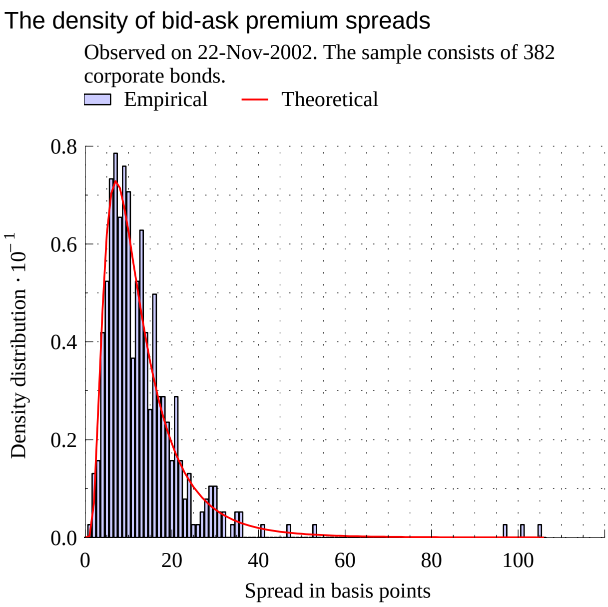

Figure 1a

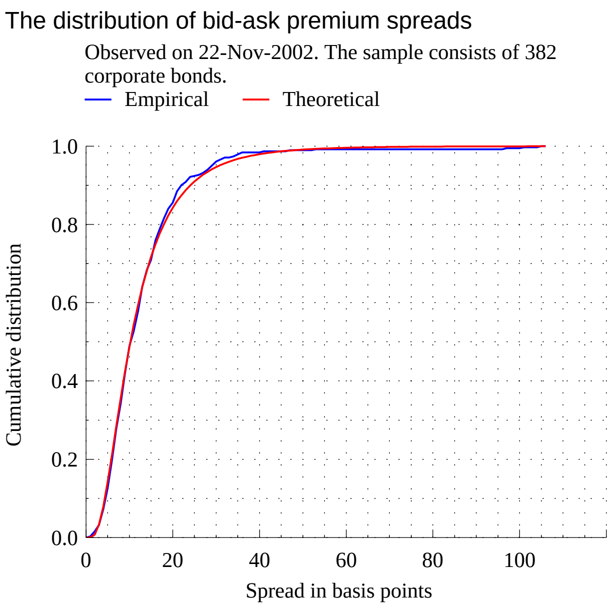

Figure 1b

Let $\Phi(x;\, \mu,\, \sigma)$ denote the lognormal cumulative distribution function with mean μ and standard deviation σ, then any corporate bond whose bid-ask premium spread, $\pi^{\text{bid-ask}}$, is larger than the critical value, π^{c}, defined by

$$\pi^{c}(\tau) = \Phi^{-1}(1 - p;\, \mu,\, \sigma) \tag{5}$$

will be excluded from the remaining sample. The probability of an outlier, p, is usually set equal to 0.01.

It is fortunate that the bond-yield based credit premia do not exhibit a significant time trend (see Figures 2a and 2b). The reason for this can be seen in the fact that the bond yield assumes a flat term structure of interest rates.

Finally, we are ready to classify the M^{*} remaining corporate bonds. Recalculate the standard deviation, σ, of the lognormal density function which is based on the remaining sample of corporate bonds. Let λ denote a "premium factor" and let η denote a "premium tolerance" such that

$$\eta = \sigma\, \lambda, \quad (\lambda \geqq 0) \tag{6}$$

The "premium tolerance" reflects the possible illiquidity effects in terms of the standard deviation of the observed bid-ask premium spreads, $\pi^{\text{bid-ask}}$.

Define a lower threshold value, denoted as θ^{low}, which is equal to the minimum of the ask-price based credit risk premia, that is,

$$\theta^{\text{low}} = \min\left(\pi^{\text{ask}}(\tau_1),\, \ldots,\, \pi^{\text{ask}}(\tau_{M^*})\right) \tag{7}$$

Next, find those corporate bonds whose ask-price based credit risk premia are less than or equal to the sum of the lower threshold value and the "premium tolerance", that is,

$$\mathcal{J}^{\text{low}} = \mathcal{J}\left(\pi^{\text{ask}}(\tau_j) \leqq \theta^{\text{low}} + \eta\right), \quad (j = 1, \ldots, M^*) \tag{8}$$

where $\mathcal{J}(\cdot)$ defines an index function which returns the ordinal number of the corporate bonds which fulfil the inequality within the round brackets and where $\mathcal{J}^{\text{low}}$ denotes the index set found, which consists of one element at least. Note that the index set consists of those bonds only, whose ask-price based credit risk premia are equal to the lower threshold value if the "premium factor" is set equal to zero.

Next, define an upper threshold value, denoted as θ^{up}, which is equal to the maximum of the bid-price based credit risk premia of those corporate bonds which belong to the index set $\mathcal{J}^{\text{low}}$, that is,

$$\theta^{\text{up}} = \max\left(\pi^{\text{bid}}(\mathcal{J}^{\text{low}})\right) \tag{9}$$

Since we seek a number of classes as small as possible, we take the maximum value in equation (9) rather than the minimum value.

In order to keep the yield curve of a particular class within the bid-ask spread, we select those bonds for the first class, whose ask-price based credit risk premia lie within the range given by the lower threshold value and the upper threshold value. Formally,

$$\mathcal{J}_1 = \mathcal{J}\left(\theta^{\text{low}} \leqq \pi^{\text{ask}}(\tau_j) \leqq \theta^{\text{up}}\right), \quad (j = 1, \ldots, M^*) \tag{10}$$

where $\mathcal{J}_1$ denotes the index set of the bonds belonging to the first class.

If it turns out that there are several classes, remove the first-class corporate bonds from the sample and repeat the procedure given by equation (1) - (10) until there are no bonds left.

2 Application

We apply the classification procedure described above to the group of bonds issued by domestic and foreign banks located in Switzerland, "Swiss banks" for short. Figures 2a and 2b show the result for a sample of 92 coupon-bearing bonds of the commercial banks in Switzerland outstanding on 15-Jan-2003 for the parameter values $\lambda = 0$ and $p = 0.01$. The rating of these bonds according to several rating agencies[5] ranges from "BBB–" to "AAA". It turns out that we get eight classes. Two bonds have been excluded due to a term to maturity which is less than one year. All the bond yields and all the credit risk premia of the remaining 90 bonds are non-negative. Nine bonds have been identified as outliers given the probability of 0.01. Of the remaining 81 bonds, 13 bonds belong to the first class, 37 bonds belong to the second class, 13 bonds belong to the third class, 1 bond belongs to the fourth class, 2 bonds belong to the fifth class, 11 bonds belong to the sixth class, 1 bond belongs to the seventh

[5] Standard & Poor's, Moody's and the Zürcher Kantonalbank (bank of the state of Zurich).

class and 3 bonds belong to the eighth class. The rating of the first-class bonds ranges from “AA+” to “AAA”, that of the second-class bonds ranges from “AA–” to “AAA”, that of the third-class bonds ranges from “AA–” to “AA+”.

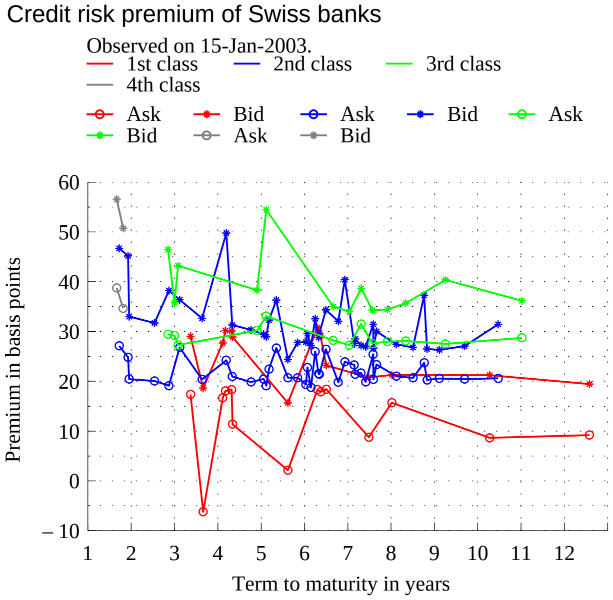

Figure 2a

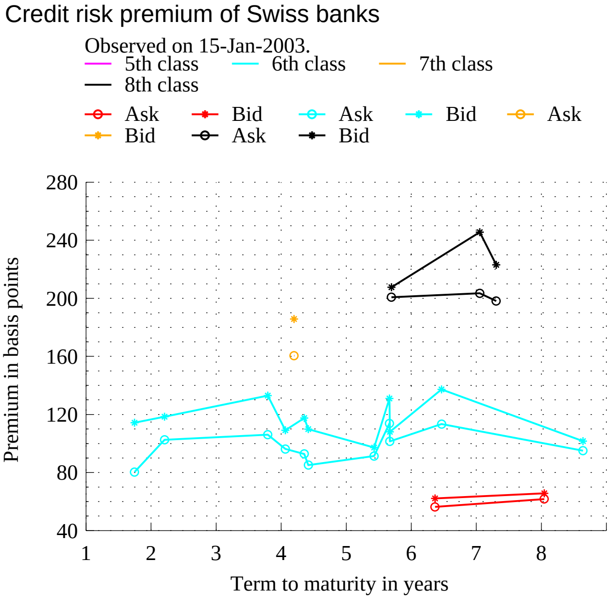

Figure 2b

In the next step, we estimate the yield curve or the term structure of interest rates, respectively, of the first three classes obtained above by means of the “forward-rate” method described in the appendix. For the estimation of the yield curve, we *assume* that the instantaneous spot interest rate of risky debt is the same as that for risk-free debt. This is not to say that we negate the possibility of a positive or negative[6] credit spread at maturity; it is rather the consequence of the fact that the zero-coupon rate of a corporate bond with a vanishing term to maturity can hardly be observed. The zero-coupon rates or spot interest rates, respectively, are shown in Figures 3a - 3c. The yield curves of the first three classes are compared with that of the government bonds in Figure 3d. We find that all the yield curves of the corporate bonds do not cross and lie above the yield curve of the government bonds. The credit spread or the credit risk premium, respectively, of the first three classes are shown in Figure 3e. A humped credit spread may be the outcome of various valuation models of risky debt[7] and empirical evidence can be found in many studies[8]. The cumulated default probalities of

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6 A negative credit spread can sometimes be observed in emerging markets, where the “sovereign ceiling” rule is violated. As reported in Durbin and Ng [2005], the credit rating agency Standard & Poor‘s made a controversial announcement in April 1997. It upgraded the debt of 14 Argentine firms, including three banks, to a rating higher than that accorded to Argentina’s sovereign debt. Consequently, Durbin and Ng [2005] found several cases of negative credit spreads for corporations with substantial export earnings and/or a close relationship with either a foreign firm or with the home government.

7 See, e.g., Longstaff and Schwartz [1995] for a structural model with a stochastic interest rate; Leland and Toft [1996] for a structural model with an endogeneous capital structure; Duffie and Lando [2001] for a structural model with incomplete accounting information.

8 See e. g. Johnson [1966]; Jones, Mason and Rosenfeld [1984]; Sarig and Warga [1989]; Fons [1994]; Helwege and Turner [1999].

the first three classes are shown in Figure 3f for two values of the recovery rate (see Hull, 2003, chapter 26).

The classification of corporate bonds is done since the beginning of the year 2003. Figures 4 - 6 show the credit risk premium of the bonds of seven groups by classes over a period of five years. The groups considered are the Cantons (federal states), the commercial banks, the mortgage institutes (Pfandbriefbanken), the industry as well as three foreign debtor groups labeled "Foreign AAA", "Foreign AA" and "Foreign A". The heterogeneous foreign debtor groups consist of bonds denominated in Swiss franc which are issued by supranational organisations, by foreign governments, states and municipalities as well as by corporations from all kinds of economic sectors. The homogeneous bonds of the mortagage institutes consist of one class only.

3 Conclusion

Our five-year experience suggests that the approach proposed in this paper works very well if, first, all the bonds considered in calculating corporate bond classes are actively traded, if, second, the sample consists of a sufficiently large number of bonds, and if, third, the terms to maturity of the corporate bonds of each class vary sufficiently.

In this paper, we have separated the task of classifying corporate bonds from the task of estimating the corresponding yield curves. We leave it to future research to find an algorithm that solves both tasks simultaneously.

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Appendix: The Estimation of the Yield Curve

In this papper, we use a convex nonlinear programming version of the "forward-rate" method proposed by Delbaen and Lorimier [1992]. Note that all the interest rates considered are meant to be continuously compounded. Let $P(\tau) \equiv P(t, t + \tau)$ denote the price of a pure discount bond whose price is fixed at time t. The discount bond is delivered at time t and matures at time $(t + \tau)$. Its term to maturity or tenor, respectively, is denoted as τ. The discount bond pays off one unit of money on the maturity day, but no coupons during its life. The corresponding spot interest rate, yield to maturity or zero-coupon rate, respectively, is denoted as $R(\tau) \equiv R(t, t + \tau)$, that is, $P(\tau) = \exp(- R(\tau) \cdot \tau)$. The function $R(\tau)$ is usually denoted as the *yield curve* or the *term structure of interest rates*, respectively. The instantaneous spot interest rate, denoted as r, is the yield of a discount bond with a vanishing term to maturity, hence $r \equiv \lim_{\tau \to 0} R(\tau) = R(0)$. Let $B(\tau) \equiv B(t, t + \tau)$ denote the cash price of a coupon-bearing bond with the same properties as a discount bond except for the coupon payments during its life time. The coupon payment on the maturity day includes the redemption value of the bond.

The ω-year forward interest rate, denoted as $F(\tau, \omega) \equiv F(t, t + \tau, t + \tau + \omega)$, corresponds with a forward contract on a pure discount bond with the agreement that the forward price is fixed at the date t and paid at the later date $(t + \tau)$ when the discount bond will be delivered. The discount bond delivered matures at the later date $(t + \tau + \omega)$. The instantaneous forward interest rate, denoted as $f(\tau) \equiv f(t, t + \tau)$, corresponds with a forward contract on a pure discount bond with a vanishing term to maturity. Hence, $f(\tau) = \lim_{\omega \to 0} F(\tau, \omega)$. It holds true that $f(0) = r = R(0)$ and $R(\omega) = F(0, \omega)$. It is well-known that the spot interest rate is equal to the average of the instantaneous forward interest rates, that is,

$$\begin{aligned} R(\tau) &= \frac{1}{\tau} \int_0^{\tau} f(t)\, \mathrm{d}t \\ P(\tau) &= \exp\big(- R(\tau)\, \tau\big) = \exp\left(- \int_0^{\tau} f(t)\, \mathrm{d}t \right) \end{aligned} \tag{A-1}$$

Suppose that a particular coupon-bearing bond will pay K coupons, c_k $(k = 0, 1, \ldots, K)$, until its maturity date. Recall that the last coupon, c_K, includes the redemption value of the bond. Then the theoretical price of a coupon-bearing bond can be written as follows.

$$B(\tau) = \sum_{k=1}^{K} c_k\, P(\tau_k) = \sum_{k=1}^{K} c_k \exp\left(- \int_0^{\tau_k} f(t)\, \mathrm{d}t \right) \tag{A-2}$$

where τ_k $(k = 0, 1, \ldots, K)$ denotes the kth cash flow period of a particular coupon-bearing bond.

Suppose that the sample under consideration consists of M bonds. Let $B^{\text{obs}}(\tau_m)$ denote the observed price of the mth coupon-bearing bond with its corresponding term to maturity τ_m, let

ϵ_m denote the tolerance of the *m*th bond constraint and T the maximum term to maturity of all the bonds in the sample, then the optimization for the "forward-rate" method in contiunuous time can be written as follows.

$$\begin{aligned}
&\min_{\{f(t)\}} \; G(T) = \int_0^T \left(\frac{\mathrm{d}f(t)}{\mathrm{d}t}\right)^2 \mathrm{d}t\,, \quad T = \max(\tau_1, \ldots, \tau_M)\,, \quad \text{subject to} \\
&\qquad \epsilon_m \geqq \left| \frac{B(\tau_m)}{B^{\text{obs}}(\tau_m)} - 1 \right|\,, \quad \text{for } m = 1, 2, \ldots, M, \\
&\qquad f(t) \geqq 0, \quad \text{for every } t.
\end{aligned} \tag{A-3}$$

The objective of the "forward-rate" method is to minimize the squared first derivative of the instantaneous forward interest rate with respect to time, that is, we seek a continuous function as smooth as possible. This minimization is subject to two constraints. The first constraint requires that the relative deviation of the theoretical price from the observed price be less than a certain tolerance. The tolerances are given by either the bid-ask spreads of bond prices or measurement errors. In a liquid market, we would like to keep the theoretical bond prices within the bid-ask spread of each bond, whereas in an illiquid market, we might use another measure which accounts for the illiquidity of the bond market. For the type of data we are using, the bid price, denoted as B^{bid}, is strictly less than the observed price which, in turn, is strictly less than the ask price, denoted as B^{ask}, that is, $B^{\text{bid}} < B^{\text{obs}} < B^{\text{ask}}$, rather than $B^{\text{bid}} \leqq B^{\text{obs}} \leqq B^{\text{ask}}$ for microstructure data. The second constraint requires the instantaneous forward interest rate to be non-negative everywhere. In a liquid market, this constraint should, in general, never be binding.

In practice, the unknown instantaneous forward interest rate is approximated by either a step function (discrete-time version; Büttler, 2000) or a series of orthogonal polynomials (continuous-time version; Büttler, 2007). In discrete time, the objective is to minimize the sum of squared first differences subject to the two constraints descibed above. In both versions, it turns out that the solution is unique and a global minimum. We showed elsewhere that the "forward-rate" method in both discrete time and continuous time is able to fully recover any simulated term structure of interest rates in a controlled environment. The discrete-time version has been used to estimate more than 23,000 yield curves of the Swiss bond markets since the beginning of 2003.

Figures

The term structure of 1st class Swiss bank bonds

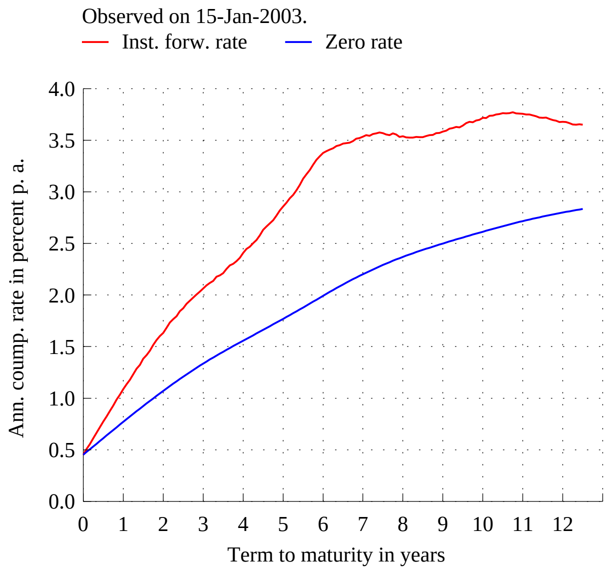

Figure 3a

The term structure of 2nd class Swiss bank bonds

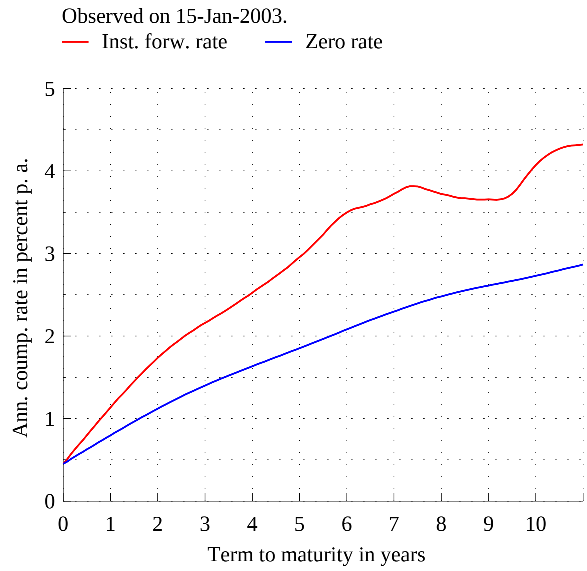

Figure 3b

The term structure of 3rd class Swiss bank bonds

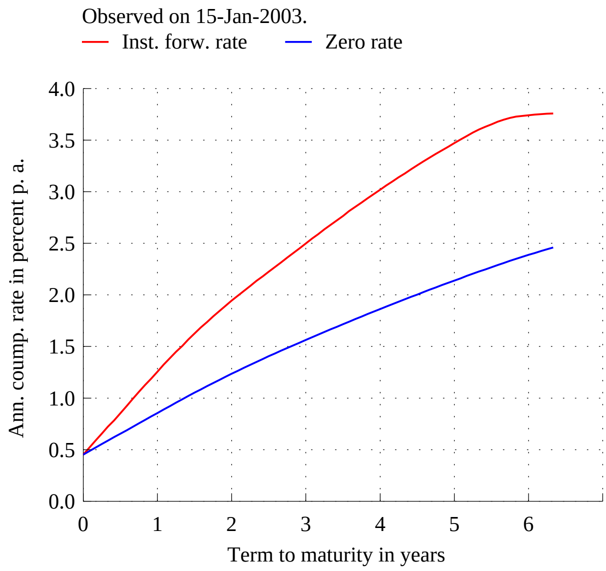

Figure 3c

The term structure of Swiss bank bonds

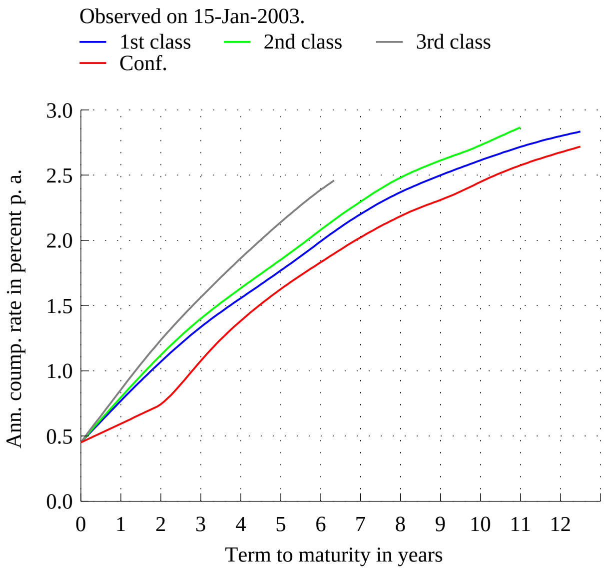

Figure 3d

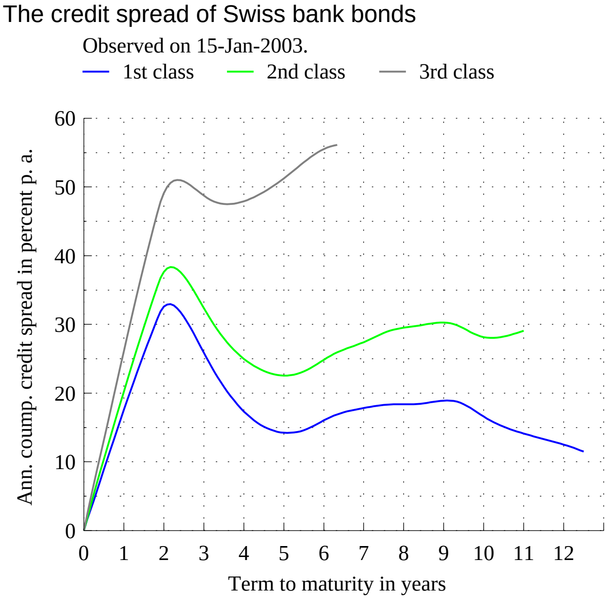

Figure 3e

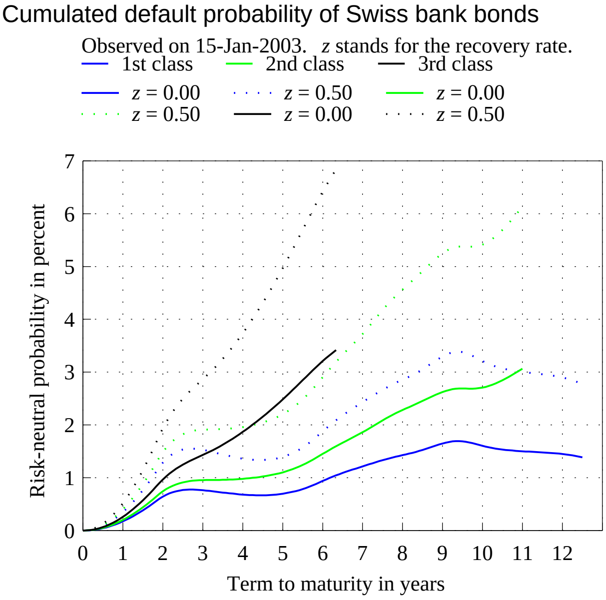

Figure 3f

The 3-year spread of Swiss first class bonds

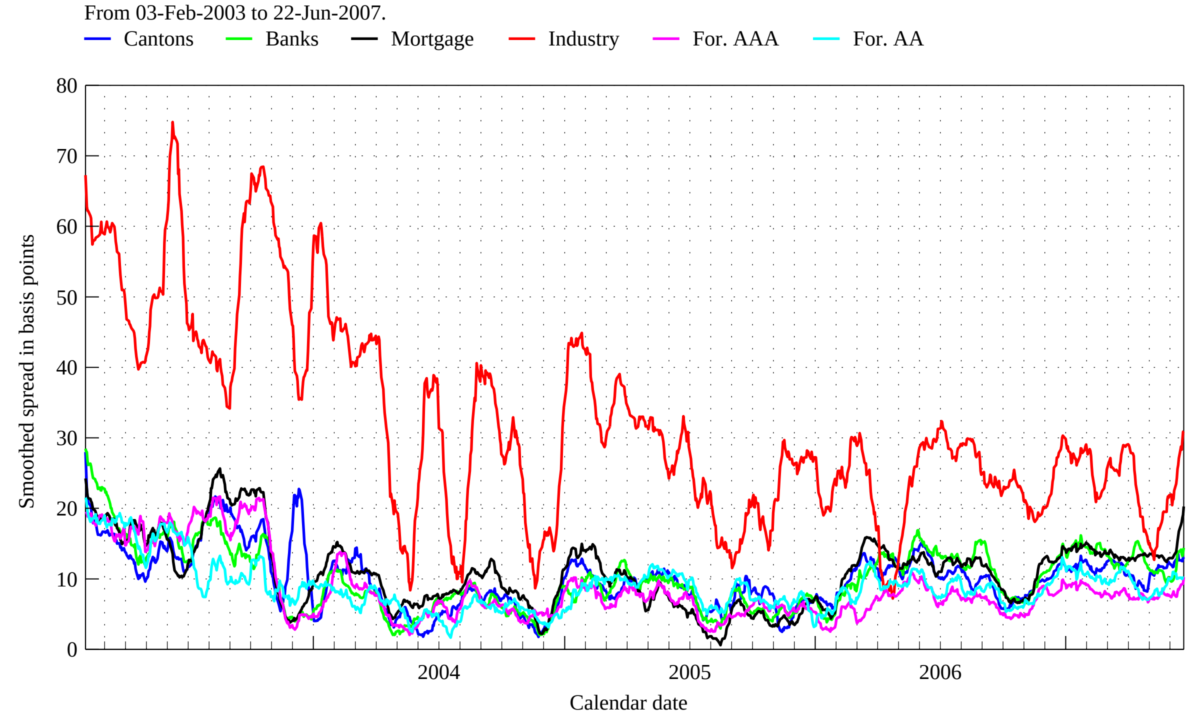

Figure 4

The 3-year spread of Swiss second class bonds

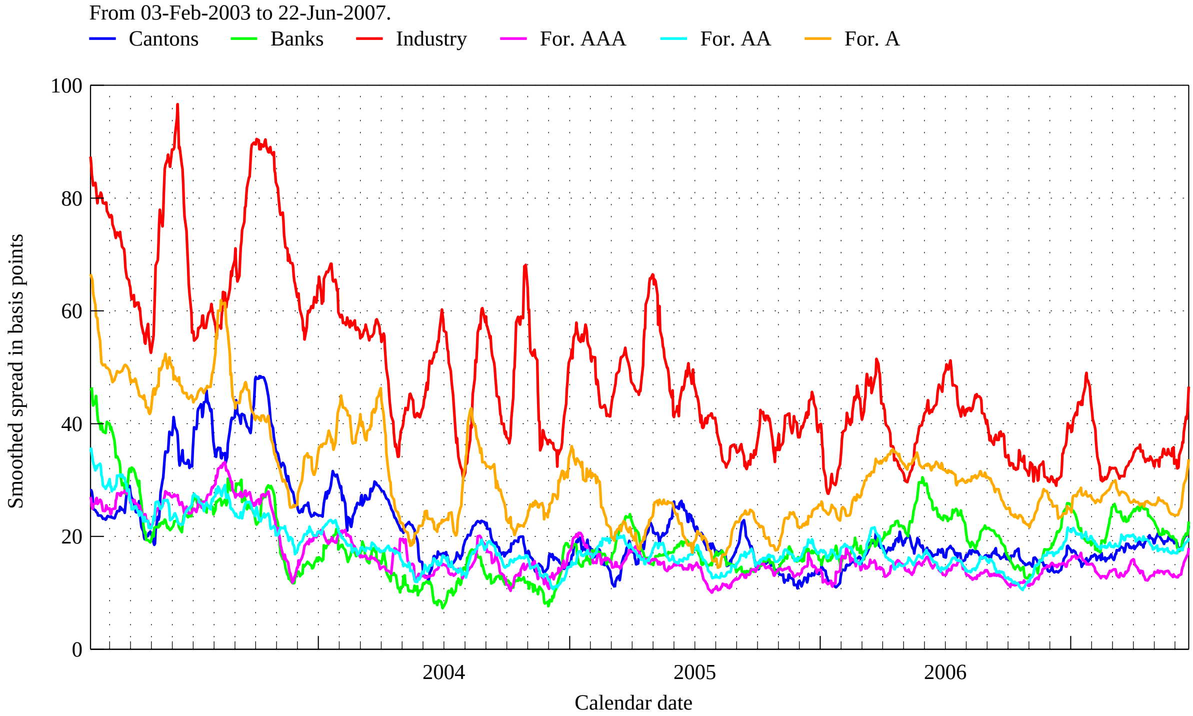

Figure 5

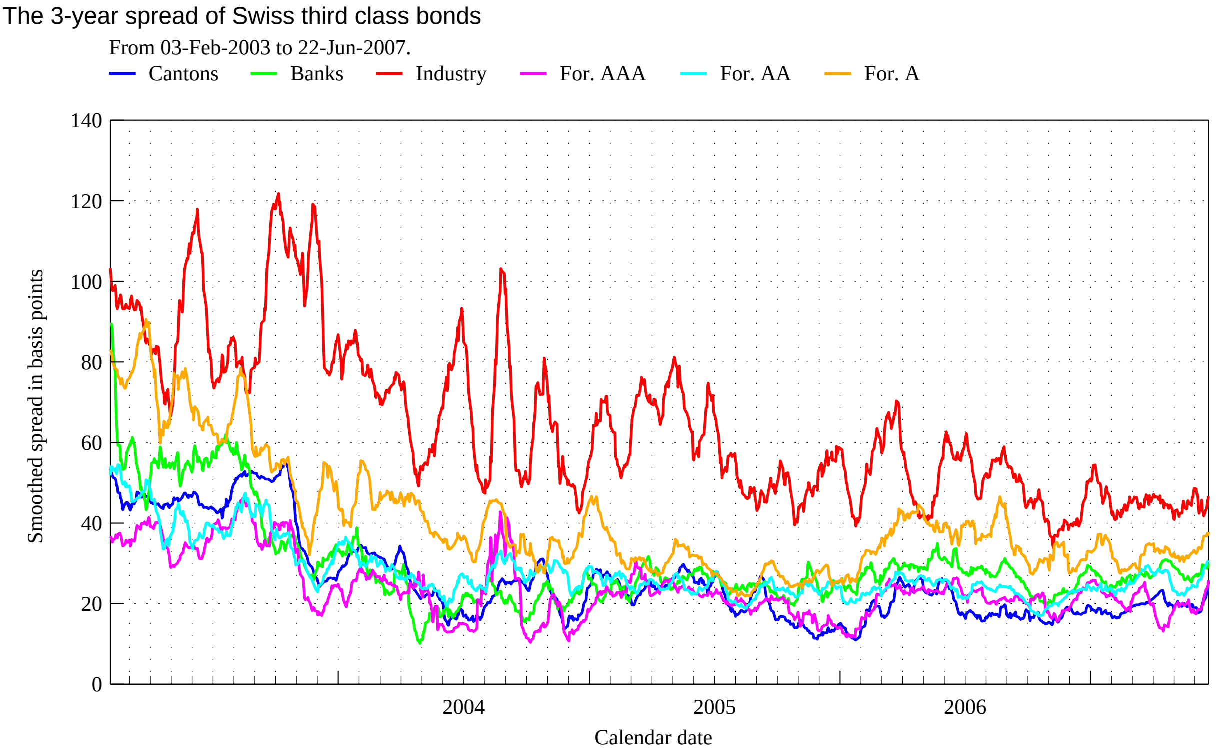
The 3-year spread of Swiss third class bonds
From 03-Feb-2003 to 22-Jun-2007.
Cantons
Banks
Industry
For. AAA
For. AA
For. A
Smoothed spread in basis points
0
20
40
60
80
100
120
140
2004
2005
2006
Calendar date

Figure 6