FREE MARKET AND LAND-USE ZONING
OF A RESIDENTIAL ECONOMY

by

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1. Introduction

It has been recognized that land-use zoning is a nearly ubiquitous phenomenon in most cities in the world. Referring to Stull (1974), the modern economics literature on land-use zoning is amazingly sparse. Moreover, there has been very little progress made in the task of incorporating zoning into the micro-economic framework of land-use models that economists are accustomed to deal with.

In this paper, we have attempted to show how land-use zoning alters the solution of the competitive housing market in equilibrium, the latter being usually treated in the "New Urban Economics". The most common items of land-use zoning are restrictions on the building height and on the building density in a given residential area. Housing is treated as a multi-dimensional good. Apart from housing space, which is regarded as the basic need for housing, three attributes of housing are considered to be the most important: the finishing of a dwelling, the garden space, and the height of a building. In principle, more attributes could be incorporated into the analysis.

The model is outlined in section 2. The solution of the free market, i.e. the competitive housing market without binding technical constraints on the design parameter of a building, is given in section 3. That of a binding restriction on the building height or on the building density is presented in section 4,the mixed constrained and unconstrained case in section 5. Finally, some conclusions are drawn in section 6.

2. The Model

The demand for housing is derived from utility maximization. The utility function is assumed to be log-linear with regard to all arguments except leisure. This choice is shown to be consistent with two empirical observations: First, the housing space demand has, approximately, unit elasticity with respect to both income and price. Second, the population density is, approximately, negatively exponential with respect to distance from the CBD. Commodities except housing, which are considered to be a composite good, leisure, housing space, and attributes of housing are arguments of the utility function. Leisure and out-of-pocket commuting cost are assumed to be linear functions in terms of distance from the CBD, i.e. traffic congestion is disregarded. The utility maximization thus becomes:

$$\max_{\{s,r\}} \quad U = \alpha_0 \ln (y-qs-k_0r) + \alpha_1 (T-kr) + \alpha_2 \ln s \pm \sum_{i=3}^{n} \alpha_i \ln h_i \tag{1}$$

where

U : utility

α_i: ($i := 0,\dots,n$) positive coefficients, the "attractions" of different consumption activities

y : household income in units of money per day

q : housing rent in units of money per area and day

s : housing space in units of area

T : nonworking hours per day

k_0: out-of-pocket commuting cost in units of money per distance k : time spent on commuting per unit of distance

r : distance from the CBD

h_i: attributes of housing. Specifically: finishing I, garden width u, and building height H

Households can choose the amount of housing space demanded and their location. In this context, location means distance from the CBD, the average trip length, however, could also be introduced. The attributes of housing are chosen by the building owner rather than by the households, but the housing rent varies with the attributes the owner decides to supply. Note that this assumption does not imply any monopoly power of the building owner as was shown in Büttler/Beckmann (1977). It is not necessary to assume that all combinations of attributes are available at every distance. We merely assume that building owners decide through profit maximization which housing attributes and what quantity thereof are to be supplied.

The supply of housing is based on profit maximization. The owner of a building is a price taker but the housing rent varies with the housing attributes supplied. The attributes of housing considered here are: the finishing of a dwelling $I \equiv h_3$, the garden width $u \equiv h_4$, and the height of the building $H \equiv h_5$. The latter has a negative marginal utility, the assumption being that tenants prefer single-storey homes to multi-storey buildings. (The negative sign in equation (1) applies to $h_5 \equiv H$.) The rental revenues per period of time are the algebraic product of housing rent and net floor surfaces. The latter are, when neglecting interior traffic areas in a building such as stairways and elevators, the number of storeys times the area of the building. Rental revenues are opposed to building cost and land cost. The land lot and the area of the building are assumed to be of a rectangular shape. The profits per period of time become in a stationary state of the residential economy:

$$\pi = q(r,I,u,H)\; F\,\frac{H}{t} \qquad \text{(rental revenues)}$$

$$- C - C_0\, F^{\varepsilon}\, H^{\gamma} \qquad \text{(structural frame cost)}$$

$$- I(i+\delta)\; D(F,H,s) \qquad \text{(finishing cost)}$$

$$- p(r)\,\{F + G(F,u)\} \qquad \text{(land cost)} \qquad (2)$$

where

π : profits in units of money per day

q : housing rent

F : area of the building

H : height of the building

t : height of a storey

C : fixed structural frame cost

C_0 : coefficient

ε,γ : structural frame cost elasticities with respect to area and height, respectively

I : cost of finishing per dwelling

i : interest rate

δ : depreciation rate

D : number of dwellings per building

p : land rent

G : garden space

u : half of garden width

The structural frame cost function is, in general, a ratio of two polynomials in terms of the height and area of the building (Büttler/Beckmann, 1977). Here it is simplified to a Cobb-Douglas function, where $\gamma > \varepsilon > 1$. The number of households or dwellings per building is defined as the floor surfaces divided by the housing space demanded per household:

$$D \equiv \frac{F\,\frac{H}{t}}{s} \qquad (3)$$

The profit and utility maximizations are interdependent through the housing attributes. The use of the design parameters in the profit maximization of equation (2) makes it possible to consider explicitly several land-use constraints and technical constraints. First of all, the number of storeys cannot be less than one. There exists a minimum area of a building $\hat{F}$ that is given for technical reasons. Two zoning regulations are considered here: The case of the building height H to be restricted to $\hat{H}$ and the case of the building density z to be restricted to $\hat{z}$. The building density is defined as the floor surfaces per lot:

$$z \equiv \frac{F \frac{H}{t}}{F+G} \tag{4}$$

Introducing the Lagrangian X and various multipliers Λ_i (i: = F, t, H, z), the function to be maximized becomes:

$$\max_{\{F,H,I,u\}} \quad X \equiv \pi + \Lambda_F(F-\hat{F}) + \Lambda_t(H-t) + \Lambda_H(\hat{H}-H) + \Lambda_z(\hat{z}-z) \tag{5}$$

If neither technical constraints nor land-use constraints are binding, then the shadow prices Λ_i are zero and, hence, the free market solution is obtained. The free-market solution will be compared to that of a binding building height, i.e. $\Lambda_H > 0$, and will be compared to that of a binding building density, i.e. $\Lambda_z > 0$.

The land rent in the long run, or in the moderate short run to be considered here, is determined by the condition that landlords absorb any profits due to location. From this it follows that building owners' profits are zero throughout the city:

$$\pi(r) = 0 \tag{6}$$

The demand for housing space and the supply of housing space are simultaneously derived from equations (1), (2), and (6) when considering various shadow prices to be zero or positive. Constant elasticity demand and supply functions are obtained.

Equilibrium on the urban housing market will now be determined under the following assumptions: All households have identical utility functions and equal income. The city population and the average income are given exogeneously. All attributes of housing are supplied at every distance. The shape of the residential zone of the city is considered to be a square ring extending between distances r_0 and r_n from the CBD. There is a rectangular road grid. Distances are then the absolute sum of East-West and North-South distances, say. The area in a strip of width dr between distances r and r+dr from the CBD is then, approximately, 4 r dr.

The supply of housing space per unit of area is $F(Ht^{-1})\ (F+G)^{-1}$ which in turn is the definition of the building density z. The demand for housing space per household is s. The ratio of housing space supplied per unit of area to housing space demanded per household is, therefore, the number of households per unit of area. This ratio is defined as the household or population density Φ:

$$\Phi \equiv \frac{F\ \frac{H}{t}}{(F+G)\, s} = \frac{z}{s} \tag{7}$$

The population density Φ is a function of the, so far, unknown housing rent q. Equilibrium on the housing market requires all households to be accomodated within the residential zone, thus:

$$P = 4 \int_{r_0}^{r_n} \Phi\ \big(q(r)\big)\ r\ dr\ , \quad r_n < \min\ (y\ k_0^{-1},\ T\ k^{-1}). \tag{8}$$

where P : city population

The equilibrium condition (8) is general in the sense that it does not depend on the particular demand and supply functions derived from equations (1), (5), and (6). It determines the overall utility level of all households with identical utility functions and equal income.

Equations (1) - (8) determine, finally, the endogeneous variables in the moderate short-run equilibrium. Note that an endogeneous city radius r_n, which is appropriate to the long run, and several income classes do not change the form of the solution presented in the following sections.

3. Free Market

The free market is the unconstrained competitive market, hence, all shadow prices in equation (5) are zero. The endogeneous variables to be considered here are: Housing rent, land rent, height and area of a building, housing space demanded per household, population density, and number of dwellings per building. In equilibrium, the relative gradient of the endogeneous variables depends on the attractions of consumption activities, structural frame cost elasticities, income, money and time commuting costs, and distance. The endogeneous variables in equilibrium are depicted in Figures 1 to 4. The land rent gradient is steeper than the housing rent gradient which is a well-known result. The building heights tend to fall with increasing distance but the converse is true for the building areas. The population density approximately falls at a negative exponential pace with increasing distance from the CBD. It is interesting to note that the number of dwellings per building is constant over the residential zone, if the out-of-pocket commuting cost is zero or comparably low. Thus, people could live in single-family homes throughout the city. This is due to three assumptions made in the model: First, constant elasticity demand and supply functions are used. Second, there is one class with equal income. Third, economies of joint occupancy through the use of traffic areas such as stairways and elevators are neglected. Introducing several income classes alters the result in the sense that poor households are squeezed into multi-family buildings, while the richest households still live in single-family homes. The garden space varies proportionally with the

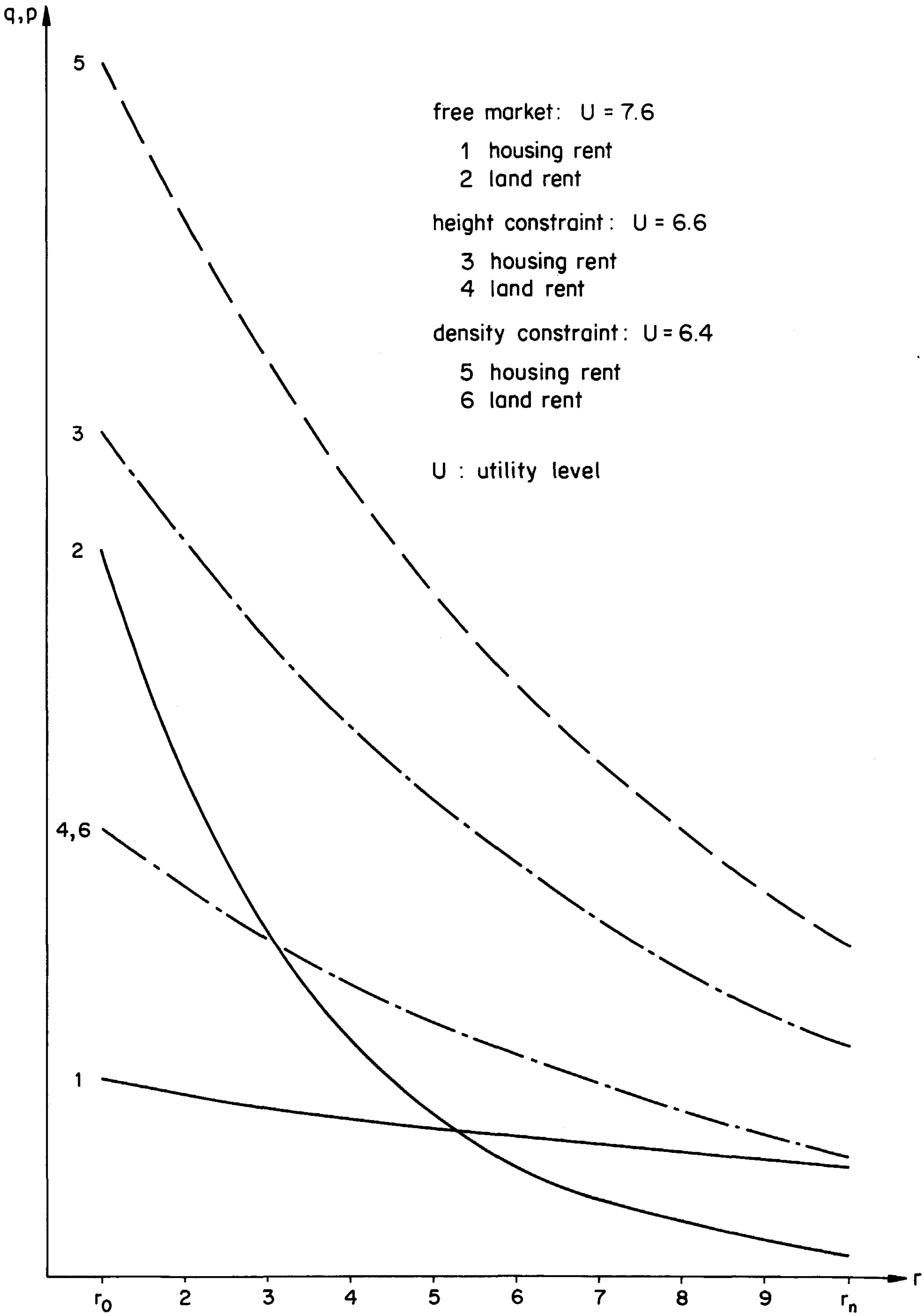

Figure 1: Housing Rent q and Land Rent p in Equilibrium

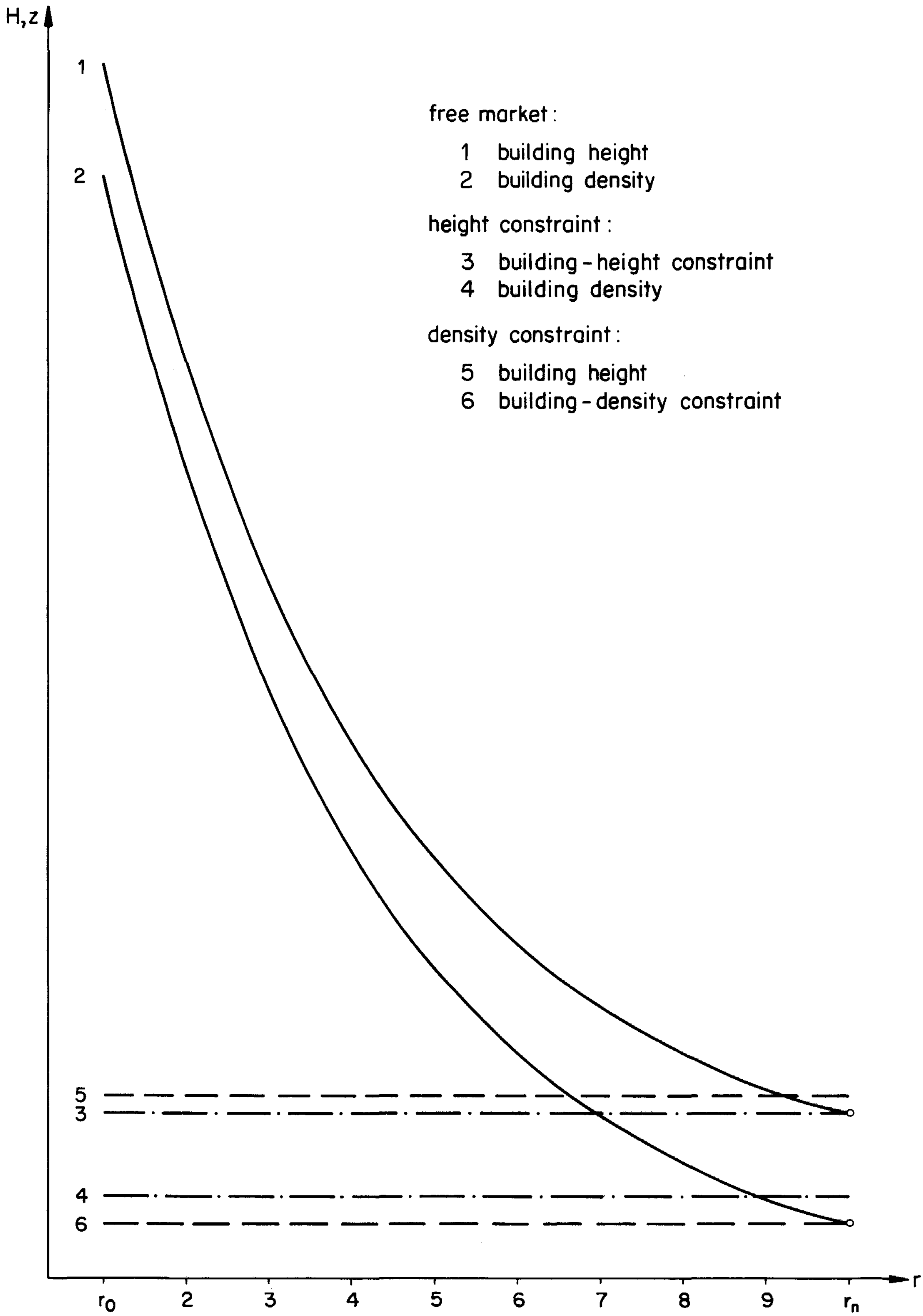

Figure 2: Building Height H and Building Density z in Equilibrium

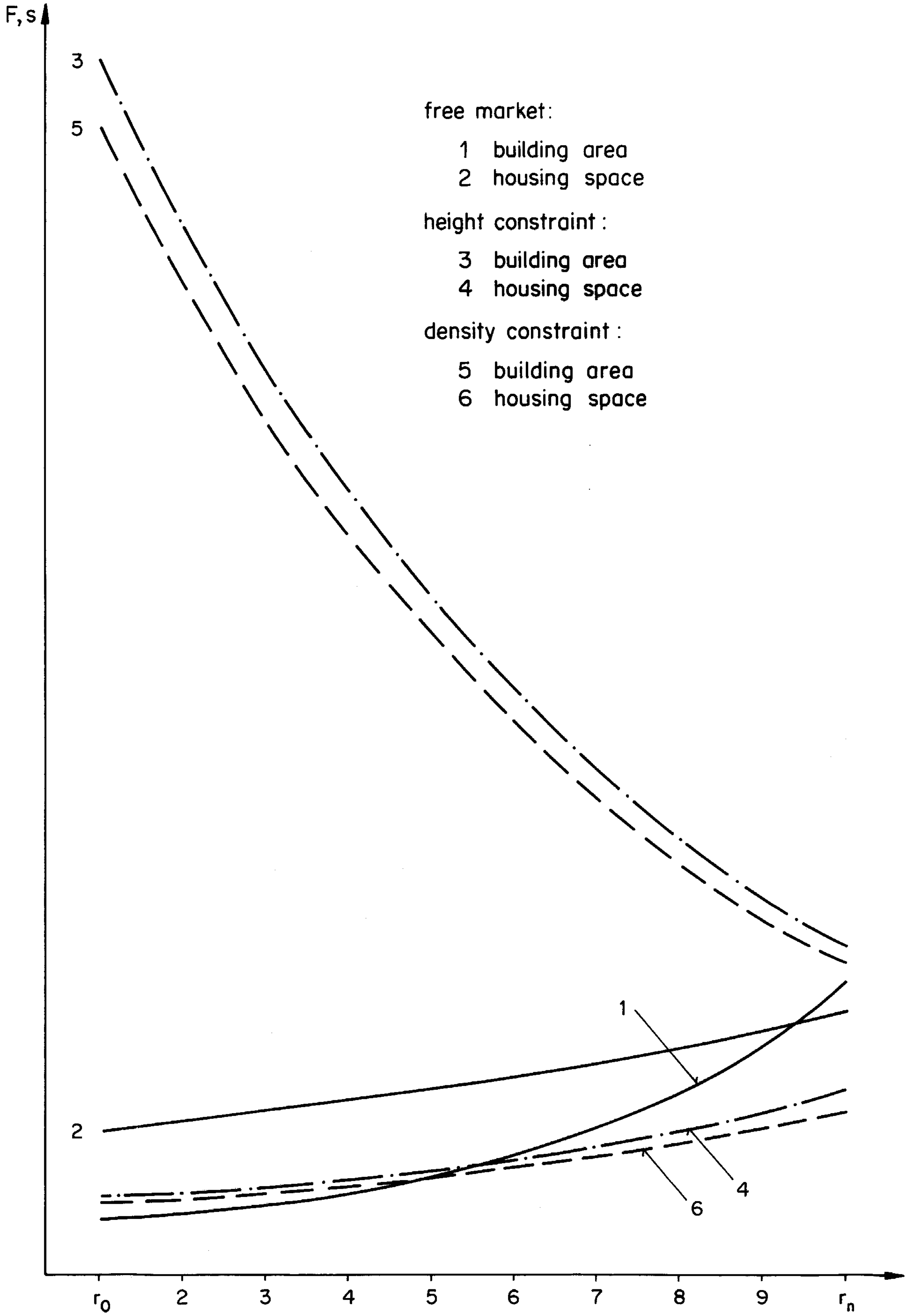

Figure 3: Building Area F and Housing Space demanded per Household s in Equilibrium

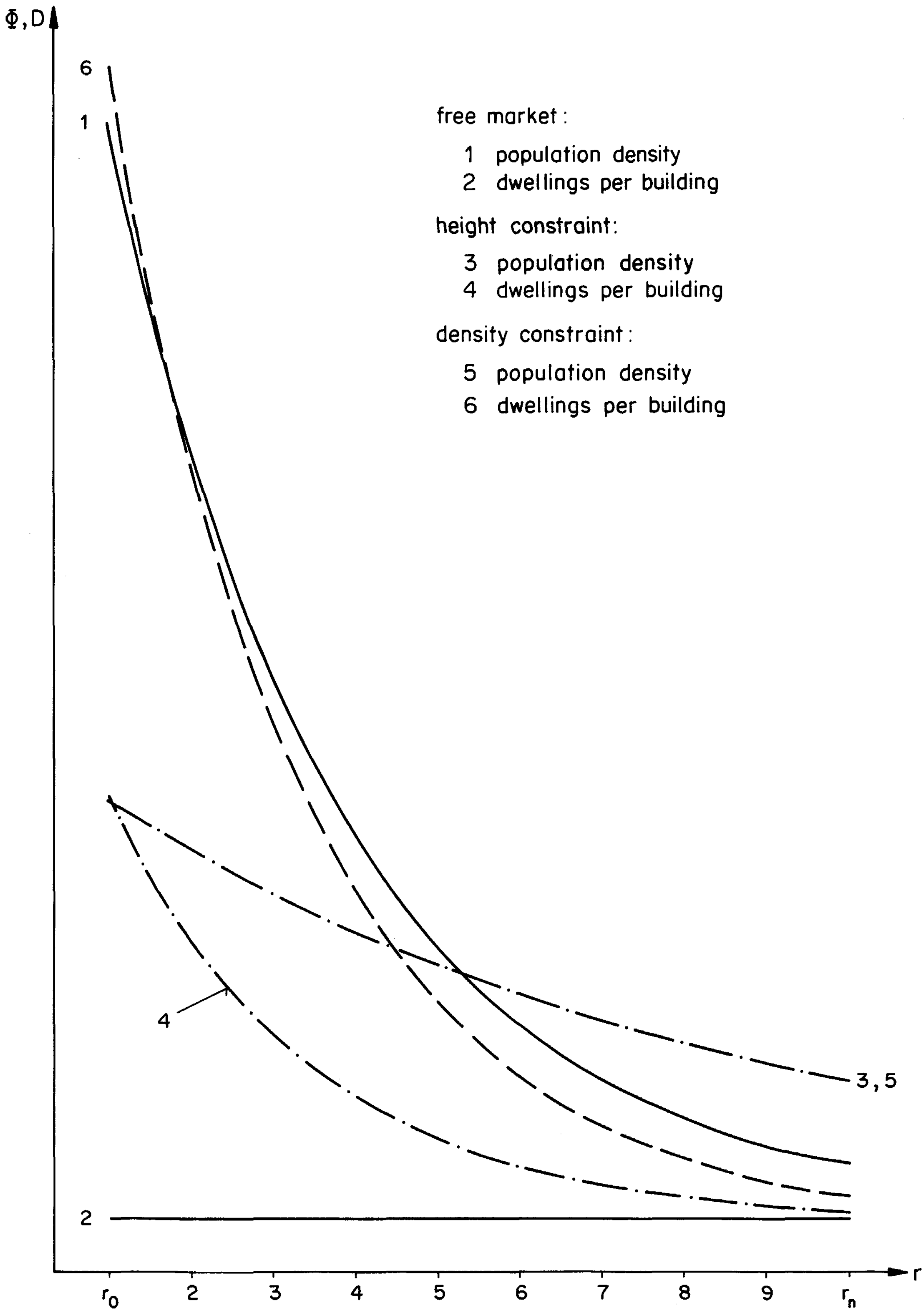

Figure 4: Population Density Φ and Number of Dwellings per Building D in Equilibrium

building area, while the finishing cost varies proportionally with the capitalized household income.

4. Land-use Zoning

Here, it is assumed that either the building-height constraint or the building-density constraint is binding throughout the city. This is visualized in Figure 2. Both cases reveal, in principle, the same result. The housing rents lie above those obtained in the case of the free market but the land rent function cuts once that obtained in the case of the free market. Moreover, in contrast to the free market the relative gradients of housing rent and land rent are both the same. This is due to the fact that in both land-use cases the building density is constant throughout the city in equilibrium. Obviously, households with identical utility functions and equal income are better off with the free market than in the case of land-use zoning. The corresponding utility numbers, as given in Figure 1, are obtained from the indirect utility functions in equilibrium. In contrast to the free market, the building areas tend to fall with increasing distance, while the building heights are constant over distance, cf. Figure 2 and 3. Moreover, the number of dwellings per building is now a decreasing function in terms of the distance from the CBD. Even in the case of one class with equal income, households could only live in single-family homes at the edge of the city. It can be shown that in the case of the free market people live more crowded near the CBD but less crowded near the edge of the city, cf. Figure 4. Similar to the free-market solution, the garden space is proportional to the building area and the finishing cost is proportional to the capitalized income.

5. Mixed Solution

The solution for a city with land-use zoning is, in general, a mixture of the single solutions presented in the previous sections. Figure 5 depicts a situation in which there are two construction zones within a residential city. The first zone extends between distances r_0 and r_2. In this zone the building heights are restricted to $\hat{H}_1$. The second zone, in which the building heights are restricted to $\hat{H}_2$, ex-

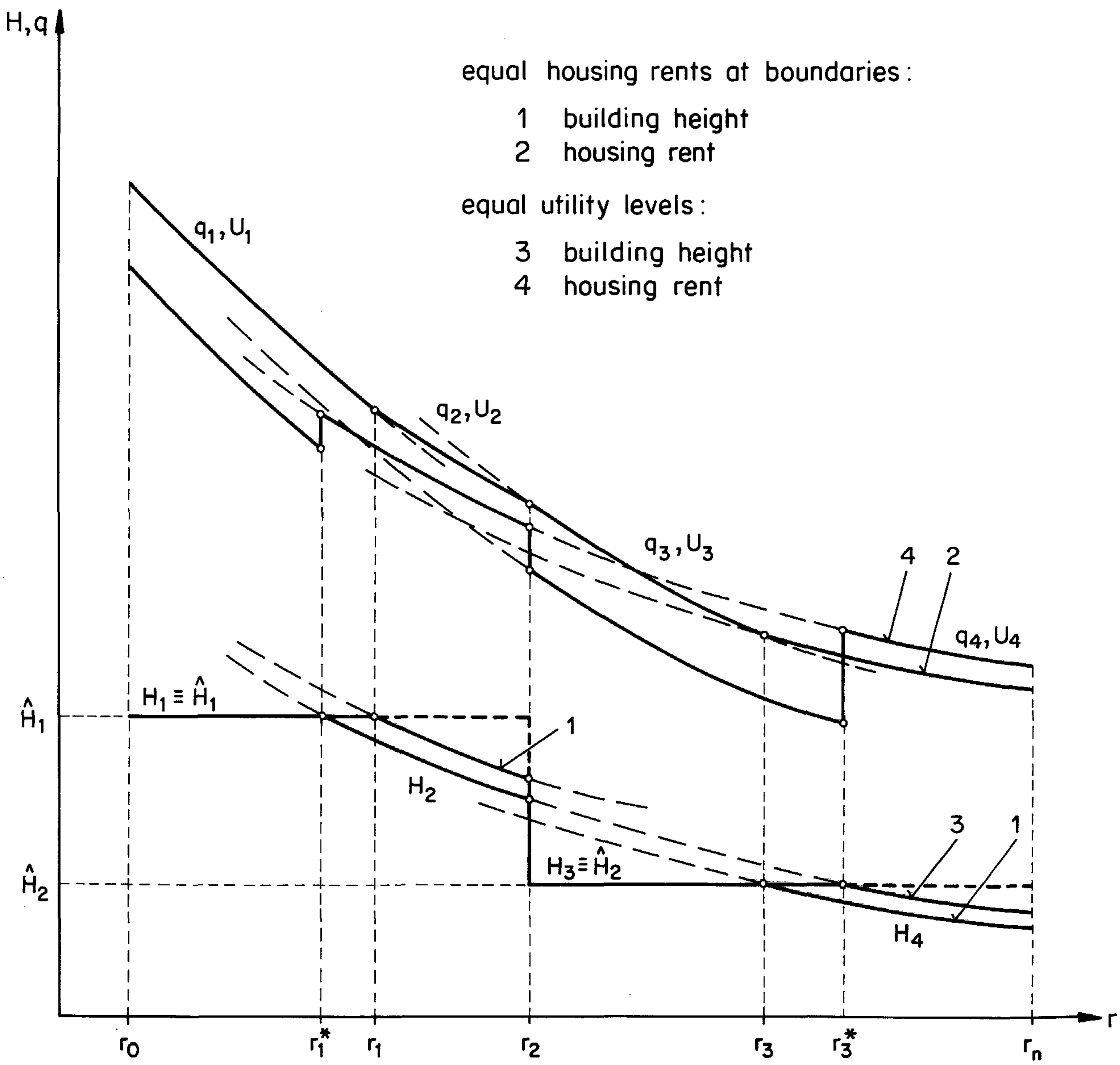

Figure 5: Mixed solution for two Construction Zones: Building Height H and Housing Rent q in Equilibrium

tends between distances r_2 and r_n. In general, the height function obtained in the case of the free market will cut these height constraints at distances r_1 and r_3 in Figure 5. Therefore, there are four zones between the distances r_0 and r_1, r_1 and r_2, r_2 and r_3, r_3 and r_n. The profit and utility maximizations are constrained in zones one and three, while unconstrained in zones two and four.

Consider the case where all households have identical utility functions and equal income. The equilibrium condition (8) has to be replaced by:

$$P = 4 \left\{ \int_{r_0}^{r_1} \Phi\left(q_1(r)\right) r\, dr + \int_{r_1}^{r_2} \Phi\left(q_2(r)\right) r\, dr \right.$$

$$\left. + \int_{r_2}^{r_3} \Phi\left(q_3(r)\right) r\, dr + \int_{r_3}^{r_n} \Phi\left(q_4(r)\right) r\, dr \right\} \tag{9}$$

The city population P is equal to the population density integrated over the four zones. The population densities are functions of the bid housing-rent functions associated with the four zones. Additional conditions are:

$$H_2(r_1) = \hat{H}_1 \tag{10}$$

$$H_4(r_3) = \hat{H}_2 \tag{11}$$

There are six unknowns to be determined: The utility levels U_i (i = 1 to 4), associated with the four bid housing-rent functions q_i (i = 1 to 4) of the four zones, and the two endogeneous zone radii r_1 and r_3. These are opposed to the three equations (9) - (11). Hence, the following proposition is obtained.

<u>Proposition</u>: Under the assumptions of the competitive housing market, an equilibrium on the housing market of a residential city with partially binding land-use constraints does not exist.

Additional assumptions are necessary to determine uniquely the equilibrium. Two "extreme" cases, which are compatible with the results of the single solutions, are considered here:

First, households bid in the same way as they would for the single solutions. Housing rents are then equal at the boundaries of the various zones:

$$q_i(r_i) = q_{i+1}(r_i) \quad , \quad i = 1 \text{ to } 3 \tag{12}$$

Equations (9) - (12) determine now the six unknowns. Assume a solution exists. It can be shown that there is an ordering of the utility levels as given below:

$$U_1 < U_2 < U_4 \tag{13a}$$

$$U_3 < U_4 \tag{13b}$$

Thus, at the end of an auction households within the fourth zone located at the edge of the city will recognize that they are better off than those within, say, the first zone located at the CBD. This is in contrast to the single solutions where all households are on the same utility level. It can be shown that the solution for the housing rent in equilibrium is a kinked curve as given as curve 2 in Figure 5. The building height function in equilibrium is given as curve 1 in Figure 5.

Second, assume households would still bid in competition within a zone but would behave like a monopsonist among zones, i.e. the utility levels associated with the four zones are equal to the overall utility level U:

$$U = U_i \quad , \quad i = 1 \text{ to } 4 \tag{14}$$

It can be shown that $U_2 < U < U_4$. The housing rent in equilibrium is now a step-wise function given as curve 4 in Figure 5. The function of the building height, given as curve 3 in Figure 5, lies between the curves 1 obtained from the first case.

It is obvious from equations (9) - (11) and (12) or (14) that all endogeneous variables are now non-homogeneous in terms of the city population, while they are homogeneous in the cases of the single solutions.

6. Conclusions

I have shown, as far as I know for the first time, how land-use zoning reveals significantly different results than those obtained in the case of the free market. Functions can be derived that allow one to differentiate between zones with binding and non-binding constraints. These functions can be tested empirically. Moreover, the mixed solution has been shown to be not determined under the assumptions of the competitive housing market. With additional assumptions, two extreme solutions have been presented. Endogeneous variables are then non-homogeneous in terms of the city population.

References:

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