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An expository note on the valuation of foreign exchange options

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This note describes the application of the binomial model to the valuation of foreign exchange options and reports simulated prices of American as well as European call and put options. We find significant deviations of simulated call prices from those reported elsewhere. This casts some doubt on empirical findings that American calls have significant pricing errors.

The modified Black–Scholes formula to value European foreign exchange options (henceforth 'FX options') has independently been derived by Garman and Kohlhagen (1983) and Grabbe (1983). In principle, they use the same argument as in the original work of Black and Scholes (1973) in that they construct a riskless hedge portfolio consisting of one call long, a certain amount of domestic bonds long, and a certain amount of foreign bonds short. Grabbe (1983) presents simulated prices of European FX call options for various parameters. These results have recently been compared with simulated prices of identical American call options in Adams and Wyatt (1987) as well as Bodurtha and Courtadon (1987). They find that the American call values have significant pricing errors. Adams and Wyatt (1987) apply the dynamic programming approach to the valuation of American call options, whereas Bodurtha and Courtadon (1987) use a numerical method proposed by Parkinson (1977).

The purpose of this note is to describe the application of the binomial approach to the valuation of FX options. The binomial model, which has been developed independently by Cox *et al.* (1979) and Rendleman and Bartter (1979), can be programmed easily and yields results that converge quite rapidly towards the theoretical values. Using the binomial model, we find significant deviations from the simulated American call prices reported in Adams and Wyatt (1987). (A comparison with those computed by Bodurtha and Courtadon, 1987, is not

* I thank Michael T. Belongia, Günter Franke, and an anonymous referee for valuable comments on an earlier, much longer draft. In a reply, Adams and Wyatt report that three simulated American call option prices were misprints. In that case, the biggest percentage difference between their and my results reduces from 8.2 per cent to 5.9 per cent. Moreover, their newly presented, simulated American put option prices also deviate significantly from mine, provided there is a premium on the American over the European put option. The largest discrepancy amounts to 6.3 per cent.

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possible because they do not present their results.) The biggest difference between the American call prices based on the binomial and on the dynamic programming approach is 8.2 per cent. In one case, the American call price reported in Adams and Wyatt (1987) is lower than that of the identical European call, which is impossible. Hence, the observed mispricing of the American FX option model might be due to inaccurate computation of the call prices.

The binomial approach to value FX call options will be presented in the next section. The two tables reported in Grabbe (1983) as well as in Adams and Wyatt (1987) will be reproduced for comparison. Finally, the binomial approach will be applied to the valuation of FX put options and similar tables will be given.

The following symbols will be used throughout:

S, S^* The current and future spot exchange rate defined as the domestic-currency price of one unit of foreign currency.

F The current forward exchange rate defined as the domestic-currency price of one unit of foreign currency to be delivered on the maturity date.

T The fixed length of calendar time to expiration of a FX option.

C, c The domestic-currency price of an American and European call option written on one unit of foreign currency.

P, p The domestic-currency price of an American and European put option written on one unit of foreign currency.

X The domestic-currency exercise price of an option written on one unit of foreign currency.

R_d, R_f The domestic and foreign riskless interest rate per fixed length of calendar time.

B_d, B_f The price of a domestic and foreign pure discount bond which matures in T units of calendar time from now. The domestic bond pays one unit of domestic currency and the foreign bond one unit of foreign currency on the maturity date.

$$B_d r_d^T = 1, \qquad B_f r_f^T = 1.$$

In the binomial model, the domestic-currency and foreign-currency amount invested in default-free bonds; *i.e.*, in a risk-free borrowing or lending.

r_d, r_f One plus the domestic or foreign riskless interest rate per fixed length of calendar time, defined as

$$r_d := 1 + R_d, \qquad r_f := 1 + R_f.$$

σ The standard deviation of the continuously compounded rate of change in the exchange rate, assumed to be constant over a fixed length of calendar time.

I. The Price of the FX Call Option

I.A. *The Binomial Model for American FX Call Options*

In our presentation of the binomial approach to the valuation of FX options, we follow that in Cox *et al.* (1979). The following *assumptions* are made:

1. In each period, the rate of change in the spot exchange rate can take on two possible values: $(u-1)$ with probability q or $(d-1)$ with probability $1-q$. The

current spot exchange rate (S) will move, at the end of the first period, to uS with probability q or to dS with probability $1-q$.
2. The rate of change in the exchange rate is greater than -1, *i.e.*, the spot exchange rate cannot be negative or zero, $u>d>0$.
3. The risk-free interest rates over a period of length h are denoted with a 'hat'. They are assumed to be constant and positive, *i.e.*, $\hat{r}_d, \hat{r}_f > 1$.
4. The logarithm of the (observed) spot exchange rate is assumed to be normally distributed. (The binomial distribution of the logarithm of the spot exchange rate relatives will converge to the normal distribution of the observed relatives.)

We introduce the ratio of the domestic interest factor to the foreign interest factor:

$$\langle 1 \rangle \qquad \hat{r} := \hat{r}_d / \hat{r}_f = (1+\hat{R}_d)/(1+\hat{R}_f),$$

and require the following inequalities to hold:

$$\langle 2 \rangle \qquad d < \hat{r} < u.$$

If these inequalities did not hold, there would be profitable riskless arbitrage opportunities. This can easily be seen when multiplying equation ⟨2⟩ by the spot exchange rate to yield $dS < S\hat{r} < uS$, which can be rewritten with the help of the interest-rate parity as $dS < F < uS$. The one-period forward exchange rate should lie in between the two possible outcomes of the spot exchange rate. For instance, suppose that $dS < uS < F$. In this case, you could make a certain profit on no investment by selling a forward FX contract. This requires no cash now. At the end of the period, you can buy FX spot and sell it at the higher forward price.

Consider one remaining period in the life of the call option. Let c be the current value of the European call, c_u be its value on the expiration date if the exchange rate goes to uS, and c_d be its value on the expiration date if the exchange rate goes to dS. On the expiration date, the call will be exercised if its exercise value $(\max[0, S^* - X], S^* := uS, dS)$ is positive; otherwise it will expire worthless. Therefore, $c_u = \max[0, uS - X]$ and $c_d = \max[0, dS - X]$. Next, we form a portfolio containing B_d units of domestic currency invested in default-free bonds (borrowing or lending) and B_f units of foreign currency invested in default-free bonds. The investment in terms of domestic currency is $B_d + SB_f$. At the end of the period, the domestic-currency denominated value of this portfolio will be

$$S^* B_f \hat{r}_f + B_d \hat{r}_d,$$

where the end-of-period spot exchange rate can take on the two possible values uS and dS. We want to choose B_f and B_d in such a way that the end-of-period values of the portfolio and the call are equal for each possible outcome, *i.e.*, we set

$$\langle 3a \rangle \qquad uSB_f \hat{r}_f + B_d \hat{r}_d = c_u,$$

$$\langle 3b \rangle \qquad dSB_f \hat{r}_f + B_d \hat{r}_d = c_d,$$

and solve for B_f and B_d,

$$\langle 4a \rangle \qquad B_f = (c_u - c_d)/(S(u-d)\hat{r}_f) \geqq 0, \qquad 0 \leqq B_f \leqq 1,$$

$$\langle 4b \rangle \qquad B_d = (uc_d - dc_u)/((u-d)\hat{r}_d) \leqq 0.$$

The portfolio containing foreign bonds long (lending) and domestic bonds short (borrowing) as given in ⟨4⟩ replicates the payoff of the call option: it is, therefore, referred to as an equivalent portfolio. In the case of the European FX call option, the current value of the call must be equal to the current value of the equivalent portfolio in order that riskless arbitrage opportunities are excluded, *i.e.*, using ⟨4⟩ the call value becomes

$$\langle 5a \rangle \qquad c = SB_f + B_d = \frac{\left(\frac{\hat{r}-d}{u-d}\right)c_u + \left(\frac{u-\hat{r}}{u-d}\right)c_d}{\hat{r}_d}.$$

Using the definition

$$\langle 5b \rangle \qquad \pi := \frac{\hat{r}-d}{u-d}, \qquad \text{implying,} \quad 1-\pi = \frac{u-\hat{r}}{u-d},$$

the value of the European call can be rewritten in the familiar form (see Cox and Rubinstein, 1985, p. 173)

$$\langle 5c \rangle \qquad c = (\pi c_u + (1-\pi)c_d)/\hat{r}_d.$$

(We use π instead of p in order to avoid confusion with the European put price.) It can be shown that the call value in ⟨5c⟩ obeys the price boundaries for European FX call options. Note that in the case of options on stocks, we only have to change the definition of π to read as $(\hat{r}_d - d)/(u-d)$ in equation ⟨5b⟩, and to reinterpret S as the price of the underlying stock.

Considering n discrete periods, the price of an American FX call option can be obtained from working backward in time from the expiration date to the current date, using for each step the one-period formula ⟨5⟩. The exercise values on the expiration date are given by

$$\langle 6 \rangle \qquad C(n,j) = \max[0, u^j d^{n-j} S - X], \qquad (j := 0, \ldots, n),$$

where $C(n,j)$ denotes the value of the call n periods from now if the spot exchange rate moved up j times. In any period, the call value is given by the larger of either the exercise value, if positive, or the value of the equivalent portfolio given in equation ⟨5c⟩:

$$\langle 7 \rangle \qquad C(n-k,j) = \max \begin{cases} u^j d^{n-k-j} S - X, \\ [\pi C(n-k+1, j+1) + (1-\pi) C(n-k+1, j)]/\hat{r}_d, \end{cases}$$
$$k := 1, \ldots, n; \qquad j := 0, \ldots, n-k.$$

In order to avoid unnecessary computations, one can eliminate those values which remain zero while working backward in time.

Since the FX option can be considered as an option on a stock paying a continuous dividend, *i.e.*, the foreign interest rate is analogous to the continuous dividend yield on the stock, the same values for the parameters u, d, h, and $\hat{r}$ can be used as in Cox and Rubinstein (1985, pp. 197–200).

I.B. An Example

The prices of European and American dollar-denominated call options written on one pound sterling have been computed for the same parameter set as in Grabbe

(1983) and Adams and Wyatt (1987). The results are reported in Tables 1 and 2. The first entry is the American call price and the entry beneath it the European call price, both obtained from the binomial model with 200 periods. For example, if the current spot exchange rate is $S = \$1.6$ per pound, the volatility of the spot exchange rate $\sigma = 0.2$, the time to expiration three months, the domestic bond price $B_d = 0.99$, the foreign bond price $B_f = 0.97$, and the exercise price $X = \$1.55$, then the American call price given in Table 2 is 7.65 cents and that of the identical European call is 7.07 cents. Adams and Wyatt (1987) find for the same American call price 7.07 cents which is exactly the European call price. The difference is 8.2 per cent. Moreover, if the time to maturity is six months, $B_d = 0.98$, $B_f = 0.94$, and $X = 1.60$, the other parameters being the same, Adams and Wyatt (1987) find a wrong American call price to be 4.44 cents, which is less than that of the identical European call. In Table 2, the European call price can be found to be 5.82 cents and

TABLE 1. Prices of American and European foreign exchange call options in cents. $S = \$1.60$, $\sigma = 0.1$, $n = 200$.

	Months			Months			Months		
	3	6	9	3	6	9	3	6	9
B_d	0.97	0.94	0.91	0.98	0.96	0.94	0.99	0.98	0.97
B_f	0.97	0.94	0.91	0.98	0.96	0.94	0.99	0.98	0.97
$X = 1.55$	6.13	7.08	7.80	6.17	7.17	7.96	6.21	7.27	8.12
	6.07	6.94	7.56	6.13	7.09	7.81	6.19	7.23	8.06
$X = 1.60$	3.11	4.30	5.15	3.14	4.37	5.27	3.16	4.43	5.39
	3.09	4.24	5.02	3.12	4.33	5.19	3.16	4.42	5.35
$X = 1.65$	1.30	2.38	3.22	1.31	2.42	3.30	1.33	2.46	3.38
	1.30	2.36	3.16	1.31	2.41	3.26	1.32	2.46	3.37
B_d	0.98	0.96	0.94	0.99	0.98	0.97	0.99	0.98	0.97
B_f	0.97	0.94	0.91	0.97	0.94	0.91	0.98	0.96	0.94
$X = 1.55$	5.50	5.95	6.23	5.15	5.32	5.41	5.52	6.01	6.34
	4.99	5.07	4.98	4.03	3.55	3.08	5.06	5.21	5.22
$X = 1.60$	2.54	3.25	3.67	2.12	2.54	2.74	2.56	3.31	3.77
	2.37	2.87	3.06	1.78	1.86	1.74	2.40	2.96	3.21
$X = 1.65$	0.97	1.62	2.05	0.72	1.12	1.33	0.98	1.66	2.12
	0.92	1.47	1.77	0.64	0.87	0.92	0.93	1.52	1.87
B_d	0.97	0.94	0.91	0.97	0.94	0.91	0.98	0.96	0.94
B_f	0.98	0.96	0.94	0.99	0.98	0.97	0.99	0.98	0.97
$X = 1.55$	7.31	9.30	11.03	8.64	11.93	15.02	7.37	9.44	11.27
	7.31	9.30	11.03	8.64	11.93	15.02	7.37	9.44	11.27
$X = 1.60$	3.97	6.07	7.86	4.98	8.26	11.34	4.00	6.16	8.01
	3.97	6.07	7.86	4.98	8.26	11.34	4.00	6.16	8.01
$X = 1.65$	1.80	3.64	5.32	2.42	5.30	8.18	1.81	3.69	5.41
	1.80	3.64	5.32	2.42	5.30	8.18	1.81	3.69	5.41

Notes: The first entry is the price of an American option and the entry beneath it is the price of the identical European option written on one pound sterling. B_d and B_f denote the price of a domestic or foreign pure discount bond, respectively. The bond prices (0.97, 0.94, 0.91), (0.98, 0.96, 0.94), (0.99, 0.98, 0.97) imply the following continuously compounded interest rates in per cent: (12.18, 12.38, 12.57), (8.08, 8.16, 8.25), (4.02, 4.04, 4.06).

TABLE 2. Prices of American and European foreign exchange call options in cents. $S = \$1.60$, $\sigma = 0.2$, $n = 200$.

	Months			Months			Months		
	3	6	9	3	6	9	3	6	9
B_d	0.97	0.94	0.91	0.98	0.96	0.94	0.99	0.98	0.97
B_f	0.97	0.94	0.91	0.98	0.96	0.94	0.99	0.98	0.97
$X = 1.55$	8.89	11.10	12.69	8.96	11.26	12.96	9.03	11.43	13.25
	8.82	10.91	12.34	8.92	11.15	12.75	9.01	11.38	13.15
$X = 1.60$	6.22	8.59	10.29	6.27	8.73	10.52	6.32	8.86	10.77
	6.18	8.47	10.04	6.24	8.65	10.37	6.31	8.83	10.70
$X = 1.65$	4.18	6.55	8.27	4.21	6.66	8.47	4.25	6.77	8.68
	4.16	6.47	8.10	4.20	6.61	8.36	4.24	6.75	8.63
B_d	0.98	0.96	0.94	0.99	0.98	0.97	0.99	0.98	0.97
B_f	0.97	0.94	0.91	0.97	0.94	0.91	0.98	0.96	0.94
$X = 1.55$	8.21	9.88	10.99	7.65	8.91	9.68	8.27	10.02	11.22
	7.92	9.27	10.01	7.07	7.80	8.02	8.01	9.50	10.41
$X = 1.60$	5.62	7.47	8.69	5.11	6.56	7.43	5.66	7.59	8.89
	5.45	7.05	7.98	4.77	5.82	6.26	5.51	7.23	8.30
$X = 1.65$	3.69	5.56	6.81	3.27	4.75	5.66	3.72	5.66	6.99
	3.59	5.29	6.31	3.09	4.28	4.86	3.64	5.43	6.57
B_d	0.97	0.94	0.91	0.97	0.94	0.91	0.98	0.96	0.94
B_f	0.98	0.96	0.94	0.99	0.98	0.97	0.99	0.98	0.97
$X = 1.55$	9.88	12.98	15.42	11.00	15.21	18.82	9.97	13.21	15.82
	9.88	12.98	15.42	11.00	15.21	18.82	9.97	13.21	15.82
$X = 1.60$	7.05	10.25	12.78	7.97	12.22	15.86	7.11	10.43	13.10
	7.05	10.25	12.78	7.97	12.22	15.86	7.11	10.43	13.10
$X = 1.65$	4.83	7.98	10.50	5.56	9.68	13.26	4.87	8.12	10.76
	4.83	7.98	10.50	5.56	9.68	13.26	4.87	8.12	10.76

Notes: The first entry is the price of an American option and the entry beneath it is the price of the identical European option written on one pound sterling. B_d and B_f denote the price of a domestic or foreign pure discount bond, respectively. The bond prices (0.97, 0.94, 0.91), (0.98, 0.96, 0.94), (0.99, 0.98, 0.97) imply the following continuously compounded interest rates in per cent: (12.18, 12.38, 12.57), (8.08, 8.16, 8.25), (4.02, 4.04, 4.06).

the American call price 6.56 cents. The difference between these two American call prices is 47.7 per cent, unless Adams and Wyatt's result is a misprint.

All European call prices given in Tables 1 and 2 are equal to those reported by Grabbe (1983), who used the modified Black–Scholes formula. With the modification mentioned in the previous section, the computer program has been used to simulate the prices of American puts written on non-dividend paying stocks, which turned out to be equal to those reported in Cox and Rubinstein (1985, Table 5-5, p. 248). In Tables 1 and 2, the foreign interest rate is equal to (greater than, less than) the domestic interest rate for the prices in the first (second, third) block. Due to the analogy with options written on continuously dividend-paying stocks, there is an incentive to exercise the call early, if the foreign interest rate is higher than the domestic interest rate. We find that, in this case, there exists a considerable premium on the American over the European call price. Significant price differences between the American call prices based on the binomial and dynamic programming approach can be found in the middle block.

I.C. Are FX Call Options Significantly Mispriced?

Adams and Wyatt (1987) studied the deviation of the theoretical from observed prices of dollar-denominated call options written on various currencies. They find that their simulated price of an option carrying a significant American premium over the identical European option lies, on average, above the observed market price. However, their simulated price of an option with a non-significant American premium is on average below the market price. We find that 147 out of 162 computed American call prices reported in Tables 1 and 2 are greater than or equal to those given in Adams and Wyatt (1987). Therefore, we expect the above-mentioned mispricing to be aggravated in the case of options with a significant American premium, but to be diminished in the case of options with a non-significant American premium. Taking all options on different currencies together, the relative pricing error appears to be of rather minor size. This is in accordance with the empirical findings in Bodurtha and Courtadon (1987, Table 4a).

II. The Value of the FX Put Option

II.A. The Binomial Model for American FX Put Options

The Black–Scholes differential equation describing the riskless hedge portfolio is equally valid for calls and puts. The only difference lies in the boundary conditions. The same must be true for the binomial approach.

Repeating the same steps as for the call, we find that foreign bonds are held short and domestic bonds are held long in the equivalent portfolio:

⟨8a⟩ $$B_f = (p_u - p_d)/(S(u-d)\hat{r}_f) \leqq 0,$$

⟨8b⟩ $$B_d = (up_d - dp_u)/((u-d)\hat{r}_d) \geqq 0.$$

The price of the European FX put must be equal to the value of the equivalent portfolio:

⟨9⟩ $$\begin{aligned} p &= SB_f + B_d \\ &= (\pi p_u + (1-\pi)p_d)/\hat{r}_d. \end{aligned}$$

where π has been defined in ⟨5b⟩. We have, therefore, the same formula as for the call given in ⟨5c⟩. Extending the one-period model to n periods as before, the price of an American FX put option can be obtained from working backward in time, using the exercise values on the expiration date

⟨10⟩ $$P(n,j) = \max[0, X - u^j d^{n-j} S], \qquad (j := 0, \ldots, n),$$

where $P(n,j)$ denotes the value of the put n periods from now if the spot exchange rate moved up j times. Again, the put value in any period is given by the larger of

TABLE 3. Prices of American and European foreign exchange put options in cents. $S=\$1.60$, $\sigma=0.1$, $n=200$.

	Months			Months			Months		
	3	6	9	3	6	9	3	6	9
B_d	0.97	0.94	0.91	0.98	0.96	0.94	0.99	0.98	0.97
B_f	0.97	0.94	0.91	0.98	0.96	0.94	0.99	0.98	0.97
$X=1.55$	1.22	2.26	3.07	1.23	2.30	3.14	1.25	2.34	3.22
	1.22	2.24	3.01	1.23	2.29	3.11	1.24	2.33	3.21
$X=1.60$	3.11	4.30	5.15	3.14	4.37	5.27	3.16	4.43	5.39
	3.09	4.24	5.02	3.12	4.33	5.19	3.16	4.42	5.35
$X=1.65$	6.21	7.20	7.96	6.25	7.30	8.11	6.29	7.40	8.28
	6.15	7.06	7.71	6.21	7.21	7.96	6.27	7.36	8.22
B_d	0.98	0.96	0.94	0.99	0.98	0.97	0.99	0.98	0.97
B_f	0.97	0.94	0.91	0.97	0.94	0.91	0.98	0.96	0.94
$X=1.55$	1.69	3.47	5.08	2.28	5.05	7.83	1.71	3.51	5.17
	1.69	3.47	5.08	2.28	5.05	7.83	1.71	3.51	5.17
$X=1.60$	3.97	6.07	7.86	4.98	8.26	11.34	4.00	6.16	8.01
	3.97	6.07	7.86	4.98	8.26	11.34	4.00	6.16	8.01
$X=1.65$	7.42	9.47	11.27	8.79	12.17	15.37	7.48	9.62	11.52
	7.42	9.47	11.27	8.79	12.17	15.37	7.48	9.62	11.52
B_d	0.97	0.94	0.91	0.97	0.94	0.91	0.98	0.96	0.94
B_f	0.98	0.96	0.94	0.99	0.98	0.97	0.99	0.98	0.97
$X=1.55$	0.91	1.54	1.95	0.67	1.05	1.26	0.92	1.57	2.02
	0.86	1.40	1.68	0.59	0.83	0.87	0.87	1.44	1.77
$X=1.60$	2.54	3.25	3.67	2.12	2.54	2.74	2.56	3.31	3.77
	2.37	2.87	3.06	1.78	1.86	1.74	2.40	2.96	3.21
$X=1.65$	5.55	6.03	6.33	5.18	5.37	5.47	5.58	6.09	6.44
	5.05	5.14	5.07	4.07	3.60	3.13	5.11	5.29	5.31

Notes: The first entry is the price of an American option and the entry beneath it is the price of the identical European option written on one pound sterling. B_d and B_f denote the price of a domestic or foreign pure discount bond, respectively. The bond prices (0.97, 0.94, 0.91), (0.98, 0.96, 0.94), (0.99, 0.98, 0.97) imply the following continuously compounded interest rates in per cent: (12.18, 12.38, 12.57), (8.08, 8.16, 8.25), (4.02, 4.04, 4.06).

either the exercise value (if positive) or the value of the equivalent portfolio given in equation ⟨9⟩:

$$\langle 11\rangle \quad P(n-k,j) = \max\begin{cases} X-u^j d^{n-k-j} S, \\ [\pi P(n-k+1,j+1)+(1-\pi)P(n-k+1,j)]/\hat{r}_d, \end{cases}$$

$$k := 1,\ldots,n; \qquad j := 0,\ldots,n-k.$$

II.B. *An Example*

The simulated prices of European and American dollar-denominated put options written on one pound sterling are reported in Tables 3 and 4 for the same parameter set used in the previous two tables. If the foreign interest rate is less than the domestic interest rate, as is the case for the prices in the bottom blocks of Tables 3

TABLE 4. Prices of American and European foreign exchange put options in cents. $S = \$1.60$, $\sigma = 0.2$, $n = 200$.

	Months			Months			Months		
	3	6	9	3	6	9	3	6	9
B_d	0.97	0.94	0.91	0.98	0.96	0.94	0.99	0.98	0.97
B_f	0.97	0.94	0.91	0.98	0.96	0.94	0.99	0.98	0.97
$X = 1.55$	3.99	6.29	7.96	4.03	6.39	8.15	4.06	6.50	8.35
	3.97	6.21	7.79	4.02	6.35	8.05	4.06	6.48	8.30
$X = 1.60$	6.22	8.59	10.29	6.27	8.73	10.52	6.32	8.86	10.77
	6.18	8.47	10.04	6.24	8.65	10.37	6.31	8.83	10.70
$X = 1.65$	9.08	11.37	13.00	9.14	11.53	13.28	9.21	11.70	13.58
	9.01	11.17	12.65	9.10	11.41	13.06	9.19	11.65	13.48
B_d	0.98	0.96	0.94	0.99	0.98	0.97	0.99	0.98	0.97
B_f	0.97	0.94	0.91	0.97	0.94	0.91	0.98	0.96	0.94
$X = 1.55$	4.62	7.67	10.11	5.32	9.30	12.77	4.66	7.80	10.36
	4.62	7.67	10.11	5.32	9.30	12.77	4.66	7.80	10.36
$X = 1.60$	7.05	10.25	12.78	7.97	12.22	15.86	7.11	10.43	13.10
	7.05	10.25	12.78	7.97	12.22	15.86	7.11	10.43	13.10
$X = 1.65$	10.09	13.29	15.82	11.24	15.58	19.31	10.19	13.53	16.22
	10.09	13.29	15.81	11.24	15.58	19.31	10.19	13.53	16.22
B_d	0.97	0.94	0.91	0.97	0.94	0.91	0.98	0.96	0.94
B_f	0.98	0.96	0.94	0.99	0.98	0.97	0.99	0.98	0.97
$X = 1.55$	3.53	5.34	6.54	3.12	4.56	5.44	3.56	5.43	6.72
	3.43	5.08	6.07	2.95	4.11	4.67	3.47	5.21	6.32
$X = 1.60$	5.62	7.47	8.69	5.11	6.56	7.43	5.66	7.59	8.89
	5.45	7.05	7.98	4.77	5.82	6.26	5.51	7.23	8.30
$X = 1.65$	8.38	10.11	11.25	7.79	9.10	9.90	8.43	10.25	11.49
	8.08	9.48	10.25	7.21	7.98	8.21	8.17	9.72	10.66

Notes: The first entry is the price of an American option and the entry beneath it is the price of the identical European option written on one pound sterling. B_d and B_f denote the price of a domestic or foreign pure discount bond, respectively. The bond prices (0.97, 0.94, 0.91), (0.98, 0.96, 0.94), (0.99, 0.98, 0.97) imply the following continuously compounded interest rates in per cent: (12.18, 12.38, 12.57), (8.08, 8.16, 8.25), (4.02, 4.04, 4.06).

and 4, there is a considerable premium on the American over the European put option, as we would expect. For example, if the current spot exchange rate is \$1.6, the volatility $\sigma = 0.1$, the bond prices are $B_d = 0.91$ and $B_f = 0.97$, the time to expiration is nine months, and the strike price $X = \$1.65$, then the American put price is 5.47 cents, whereas that of the identical European put is 3.13 cents, *i.e.*, there is a premium of 74.8 per cent (see Table 3). An interesting case arises for the same parameter set changing only the time to expiration. When the time to expiration is varied from three to nine months with a three-month step, the price of the European put option decreases from 4.07 to 3.60 to 3.13 cents (see bottom line in Table 3), whereas the price of the identical American put option increases. The price of the American put option must increase as the time to expiration increases, other things being equal. In the above comparison, however, the interest rates are not held constant, while the time to expiration is varied.

References

ADAMS, PAUL D., AND STEVE B. WYATT, 'On the Pricing of European and American Foreign Currency Call Options,' *Journal of International Money and Finance*, September 1987, **6:** 315–338.

BLACK, FISCHER, AND MYRON SCHOLES, 'The Pricing of Options and Corporate Liabilities,' *Journal of Political Economy*, May 1973, **81:** 637–654.

BODURTHA, JAMES N. JR., AND GEORGES R. COURTADON, 'Tests of an American Option Pricing Model on the Foreign Currency Options Market,' *Journal of Financial and Quantitative Analysis*, June 1987, **22:** 153–167.

COX, JOHN C., STEPHEN A. ROSS, AND MARK RUBINSTEIN, 'Option Pricing: A Simplified Approach,' *Journal of Financial Economics*, September 1979, **7:** 229–263.

COX, JOHN C., AND MARK RUBINSTEIN, *Options Markets*. Englewood Cliffs, NJ: Prentice-Hall, 1985.

GARMAN, MARK B., AND STEVEN W. KOHLHAGEN, 'Foreign Currency Option Values,' *Journal of International Money and Finance*, December 1983, **2:** 231–237.

GRABBE, J. ORLIN, 'The Pricing of Call and Put Options on Foreign Exchange,' *Journal of International Money and Finance*, December 1983, **2:** 239–253.

PARKINSON, MICHAEL, 'Option Pricing: The American Put,' *Journal of Business*, January 1977, **50:** 21–36.

RENDLEMAN, RICHARD J., AND BRIT J. BARTTER, 'Two-State Option Pricing,' *Journal of Finance*, December 1979, **34:** 1093–1110.