1. Formulating the hypothesis

1.1. Introduction

In recent years, the money stock has become increasingly important as a target variable of monetary policy. Following the changeover to flexible exchange rates, the Swiss National Bank too began to gear its monetary policy to a specific rate of growth of the money stock (cf. Schiltknecht). To do this, a mechanism for controlling money stock growth had to be developed. We will discuss below models that can be used to control the money stock in the short run. Our starting point is the definition of the money multiplier,

\[ M = m \cdot B, \]  

where \( M \) = money stock narrowly defined, \( m \) = money multiplier, and \( B \) = monetary base.

It is assumed that a specific stock target \( M^* \) is fixed for each month. If the money multiplier can be correctly predicted, the monetary base \( (B^*) \) needed to achieve the money stock target is

\[ B^* = M^*/\hat{m}, \]  

where \( \hat{m} \) = money multiplier predicted.

According to eq. (2), for a specified money stock target \( M^* \) to be achieved it must be possible to predict as accurately as possible the money multiplier
and to use the monetary base as the National Bank's control variable. Consequently, it is necessary to start from a monetary base over which the National Bank has the tightest possible control.

The strategy of predicting the money multiplier for the purpose of controlling the money stock has been developed in recent years by Burger, Kalish and Babb (1971) and by Burger (1972) for the United States and by Bomhoff (1977) for the United States and the Netherlands. No one has yet attempted to do the same for Switzerland. To predict the money multiplier in the United States, Burger, Kalish and Babb use the following equation:

\[
m_t = b_0 + b_1 \cdot X_{1t} + b_2 \cdot X_{2t} + \sum_{i=1}^{11} b_{i+2} \cdot d_i + \rho u_{t-1},
\]

where

\[
m = \text{money multiplier predicted},
\]
\[
X_1 = \text{average of the three previous money multipliers},
\]
\[
X_2 = \text{factor allowing changes in the statutory reserve requirements},
\]
\[
d_t = \text{dummy variable to allow for the seasonal factor},
\]
\[
\rho u_{t-1} = \text{adjustment factor for autocorrelation of the residuals}.
\]

Burger (1972) has modified this equation by incorporating as an additional variable the percentage change in the Treasury bill rate in the preceding period and dropping the variable that allows for changes in the reserve requirements. Bomhoff (1977) adopts a different approach. For the purpose of predicting the multiplier, he used a Box-Jenkins moving-average model.

Other approaches to money stock control, such as those adopted by Schadrack and Skinner (1972) or by Hamburger (1972), are also based on a single equation yet explain the money stock directly. These approaches will be disregarded, and no attempt will be made to formulate a comprehensive money-market model. An account of various models developed in the United States and their predictive properties can be found in the article by Levin (1973).

Of the US models mentioned, the one developed by Bomhoff (1977) gives the best forecasting results for the period of fixed exchange rates. By contrast, an empirical test of the adequacy of Bomhoff's approach for Switzerland has proved negative because the money multiplier cannot be regarded as a stationary stochastic process. This suggests that the approaches developed for the United States cannot be directly applied to the situation in Switzerland.

The following section discusses some of the features peculiar to the money supply in Switzerland and establishes the hypothesis for the model used to predict the multiplier. The second half of this paper will describe the models estimated and their predictive properties.
1.2. Features peculiar to the money supply in Switzerland

When discussing the money supply in Switzerland, it is best to start from the definition of the monetary base. In the Report ‘Revision der Geldmengenstatistik’ (1975), the source of the monetary base is defined in the following manner:

\[ B = IR + OM + RF + U, \]

where
- \( B \) = monetary base,
- \( IR \) = foreign exchange reserves,
- \( OM \) = open market portfolio (own securities),
- \( RF \) = advances to commercial banks,
- \( U \) = other assets less other liabilities on the balance sheet of the National Bank, particularly the time liabilities of banks.

The foreign exchange reserves held at the end of a given period may be written as follows:

\[ IR = IR_{-1} + EB - NKEB - NKEP - NKEG, \]

where
- \( IR_{-1} \) = foreign exchange reserves at the end of the preceding period,
- \( EB \) = current account balance; (+) denoting surplus; (—) denoting deficit,
- \( NKEB \) = banks' net capital exports,
- \( NKEP \) = non-banks' net capital exports,
- \( NKEG \) = Federal Government's net capital exports.

Inserting (5) in (4) we obtain

\[ B = IR_{-1} + EB - NKEB - NKEP - NKEG + OM + RF + U. \]

Use of the monetary base, on the other hand, is expressed as follows:

\[ B = R + BR + FR + C, \]

where
- \( C \) = currency circulating in the private non-bank sector,
- \( R \) = banks' minimum required reserves,
- \( BR \) = banks' borrowed reserves (\( BR = RF \)),
- \( FR \) = banks' free reserves.

In addition to the monetary base, thus defined, the ‘adjusted’ monetary base is used in order to analyse money supply processes. The adjusted
monetary base \((BMB)\) is obtained by deducting advances to commercial banks \((RF)\) from the monetary base, i.e.,

\[
BMB = IR_{-1} + EB - NKEB - NKEP - NKEG + OM + U = R + FR + C. \tag{8}
\]

In the analysis below, the adjusted monetary base is used as it has been under the direct control of the Swiss National Bank since the changeover to flexible exchange rates. Another reason for choosing the adjusted monetary base is that advances to the banks have played only a minor role as a source of variation in the Swiss monetary base. Exceptions to this rule are the period from January 1974 to mid-1975 and the end-of-quarter borrowing by banks for window-dressing purposes. Under fixed exchange rates, any expansion in the monetary base was, in practice, due entirely to an increase in foreign exchange reserves. The introduction of flexible exchange rates has not altered this in any way since open-market operations are seldom employed.

The monetary base is subject to substantial short-run variability. This is due to short-term capital movements triggered by currency speculation and by political uncertainties arising in the short run. Since, apart from seasonal fluctuations, currency and minimum required reserves, in the short-run, are stable entities and since borrowed reserves are a very minor item, short-term capital movements have affected mainly the level of free reserves \((FR)\). Such fluctuations occur even under a system of flexible exchange rates as a result of substantial central bank intervention in the foreign exchange market. In this context, it is necessary to investigate how the money stock, in general, and the share supplied by the banks, in particular, react to such short-run changes in the monetary base.

The money stock would adjust instantaneously to a change in the monetary base if economic agents had full information and adjustments to the desired portfolios were costless. It is obvious that both conditions are not fulfilled. For this reason, banks must form expectations about the future trend of the monetary base. These expectations might be the outcome of an evaluation process as to what extent a change in the monetary base is due to a deterministic or stochastic Central Bank policy. It is assumed that money supply adjustments occur only in response to deterministic or permanent changes in the monetary base and that stochastic or transitory changes are absorbed by a variation in the reserves of the banking system.

Let the actual growth rate of the monetary base \(\dot{B}/B\) be the sum of a permanent component \(\dot{B}^p/B^p\) and a transitory component \(\varepsilon\),

\[
\dot{B}/B = \dot{B}^p/B^p + \varepsilon. \tag{9}
\]
Similarly, the actual growth rate of the money stock consists of a permanent component \( \dot{M}^P/M^P \) and a transitory component \( \mu \),

\[
\frac{\dot{M}}{M} = \frac{\dot{M}^P}{M^P} + \mu.
\]  

(10)

It is assumed that the transitory parts \( \epsilon \) and \( \mu \) have zero means, as well as constant variance, and are independent temporally and of each other.

The actual growth rate of the money stock can be expressed as the sum of the growth rates of the money multiplier and the monetary base,

\[
\frac{\dot{M}}{M} = \frac{\dot{m}}{m} + \frac{\dot{B}}{B}.
\]  

(11)

Similarly, we define the same relationship for the permanent parts of eq. (11),

\[
\frac{\dot{M}^P}{M^P} = \frac{\dot{m}^P}{m^P} + \frac{\dot{B}^P}{B^P}.
\]  

(12)

Combining (10)-(12), the actual growth rate of the money multiplier is obtained,

\[
\frac{\dot{m}}{m} = \frac{\dot{m}^P}{m^P} + \frac{\dot{B}^P}{B^P} - \frac{\dot{B}}{B} + \mu.
\]  

(13)

The problem now arises how banks form their expectations about the permanent growth rate of the monetary base. In the past it was not customary to announce specific targets for the money stock. Furthermore, one can assume that behaviour has not yet been altered fundamentally by the adoption of a money stock target. Therefore, banks are assumed to rely on past and current information to form their expectation. A linear combination with variable coefficients is chosen,

\[
\left( \frac{\dot{B}^P}{B^P} \right)_t = \sum_{j=0}^{i} \beta_j (\dot{B}/B)_{t-j}.
\]  

(14)

If eq. (14) is a minimum-variance estimator, it can be shown that the weights and, therefore, the speed of adjustment depend on the relative variance of the permanent and transitory increments of the monetary base [cf. Lucas (1976)]. Since in the past the short-run fluctuations in the monetary base were large relative to the growth rate, long adjustment periods are expected to prevail. An analogous combination for the change in the permanent money multiplier \( \dot{m}^P/m^P \) is assumed. Substituting this and eq. (14) into eq. (13) and rearranging terms, the equation to be estimated is obtained:

\[
\frac{\dot{m}_t}{m_t} = \sum_{i=1}^{k} \frac{\alpha_i}{1 - \alpha_0} \frac{\dot{m}_{t-i}}{m_{t-i}} + \sum_{j=1}^{i} \frac{\beta_j}{1 - \alpha_0} \frac{\dot{B}_{t-j}}{B_{t-j}} + \frac{1}{1 - \alpha_0} \frac{\dot{B}_t}{B_t} + \mu.
\]  

(15)
2. Empirical results

The hypothesis as formulated in eq. (15) was tested with monthly data for the period January 1950 to August 1976. The equations were estimated using a generalized form of a Box-Jenkins transfer function. A description of the estimation technique and a detailed discussion of the results can be found in the appendix.

Several equations using different lag structures were estimated. All of them [see eqs. (16), (17) and (18) of the appendix] confirmed the expected sluggish adjustment of the money stock to a change in the adjusted monetary base. An evaluation of the various equations showed that the number of autoregressive terms could be reduced without a substantial loss of forecasting power. The best forecasting properties were obtained with eq. (17) of the appendix. In this equation the time lags for the monetary base are 0, 3, 6, 9, and 12 months, while the time lags of the money multiplier are 1, 6, and 12 months. Figs. 1 and 2 depict the standard deviations of the percentage deviation and the 90\% region of confidence of the forecasts of eq. (17), respectively. These figures were obtained from multi-steps-ahead forecasts. They show that the prediction process is stable and the confidence region is quite narrow. Fig. 2 shows that 90 percent of all predictions carried out for, say, a period six months hence were subject to errors varying only between \(-3.5\) and \(+4.5\) percent. In order to test the forecasting power of the model, eq. (17) was estimated for the period January 1950 to December 1974. Ex-ante predictions are then computed for the next 20 periods, assuming that the adjusted monetary base was known. This assumption is justifiable since the Swiss National Bank has direct control over the adjusted monetary base. The predictions for the 20 months ahead are shown in table 1. The
biggest prediction error occurred after the eleventh month and was about 2 percent.

It is not clear how the transmission from a fixed to a flexible exchange rate system has altered the money supply process. For instance, Bomhoff (1977) claims that for the Netherlands a shift in the structure of the money multiplier occurred after 1969 because his predictions of the money multiplier deteriorated sharply in 1969. To test the stability of the money multiplier model for Switzerland, the observation period was partitioned into three subperiods (1950–59, 1960–69, and 1970–76). From table 2 of the appendix it can be seen that the coefficients do not vary substantially and are, therefore, quite stable. It seems that the transition from fixed to flexible exchange rates has not altered the transmission process from the monetary base to the money stock.

3. Conclusions

The results of the empirical study indicate that the money multiplier can be reliably predicted. From a technical point of view the control of the money stock should become easier in the future. The results also show that,
in the very short run, changing the monetary base has little impact on the money stock. The sluggish adjustment of the money stock to changes in the monetary base means that, if money stock growth deviated from the target, this could be corrected in the short run only by a drastic change in the adjusted monetary base. This might, in the short run, disrupt the money and foreign exchange markets and, in the long run, induce a deviation in the opposite direction from the money stock target. The monetary base must therefore be corrected in small steps. This is the only way to ensure that the money stock growth target will be achieved in the long run and that the money stock will not fluctuate sharply around the set path.

Appendix 1

A.1. Concepts and data sources

The concepts used below have been abbreviated as follows:

\( t \)       Period (month).
\( \text{BMB}_t \) Adjusted monetary base in month \( t \).
\( M1_t \)    Money stock, narrowly defined, in month \( t \).
\[ m_t = \frac{BMB}{M1_t} \]

Multiplier in month \( t \).

\[ x_t = \frac{BMB_t - BMB_{t-1}}{BMB_{t-1}} \]

Percentage growth in the adjusted monetary base in month \( t \).

\[ y_t = \frac{m_t - m_{t-1}}{m_{t-1}} \]

Percentage increase in the multiplier in month \( t \).

\[ e_t = \hat{m}_t - m_t \]

Prediction error in month \( t \).

\[ AC_x(k) \]

Autocorrelation coefficient of series \( \{x\} \) for time lag \( k \).

\[ CC_{xy}(k) \]

Cross-correlation coefficient for series \( \{x\} \) and \( \{y\} \) for time lag \( k \).

The empirical study employs data on the adjusted monetary base \( \{BMB\} \) and the narrowly defined money stock \( \{M1\} \), covering the period from January 1950 to August 1976 (320 observations). The data for the money stock correspond with the figures compiled by the Swiss National Bank. The data for the adjusted monetary base, however, are derived from unpublished data collected by the Swiss National Bank. In order to be able to calculate some form of monthly average (average of the figures from the returns published by the National Bank), the adjusted monetary base has been defined as the sum of notes and coins in circulation and balances at the National Bank less the balances of foreign central banks and other banks, the Federal Government’s frozen balances and advances to commercial banks.

\subsection*{A.2. Forecasting models}

On the basis of the hypothesis formulated in section 1.2, the following class of models has been examined:

\[ \hat{y}_t = \sum_{i=1}^{r} \alpha_i y_{t-L_i} + \sum_{j=1}^{q} \beta_j x_{t-L_j} + \sum_{k=1}^{r} \gamma_k (\hat{y}_{t-L_k} - y_{t-L_k}) + \delta, \]

\[ L_i > 0, \quad L_j \geq 0, \quad L_k > 0, \]

\[ p \geq 0, \quad q \geq 0, \quad r \geq 0, \]

and derived from this,

\[ \hat{m}_t = m_{t-1}(1 + \hat{y}_t). \]
In the study the following three steps have been taken:

1. Estimating the parameters \( \{\alpha\}, \{\beta\}, \{\gamma\}, \) and \( \delta \) on the basis of the available data by minimizing the sum of the squares of residuals \( (m_t - \hat{m}_t) \).

2. Testing the residuals in the manner suggested by Box-Jenkins (1970, chs. 10, 11)\(^1\)
   - testing their distribution function,
   - examining their autocorrelation functions,
   - testing the model hypothesis by means of Chi-square-tests.

3. If the residuals satisfy the requirements formulated in appendix 3 the percentage deviations of the 'ex-post' one-step-ahead predictions have been calculated. \([\text{Percentage deviation} = 100 \cdot (m_t - \hat{m}_t)/m_t]\]

A.3. Models of the class \( p > 0, q > 0, r > 0 \)

These models correspond roughly\(^2\) to the transfer-function model constructed by Box-Jenkins. \( L_0, L_j, \) and \( L_k \) are chosen by means of the autocorrelation and cross-correlation coefficients. If not otherwise indicated, the observation period is February 1951 to August 1976.

\[
L_t = 1, 2, 3, 4, 5, 6, 8, 9, 11, 12, \quad p = 10, \quad \begin{array}{c} L_j = 0, 1, 2, 3, 5, 6, 8, 9, 11, 12, \quad q = 10, \end{array} \quad \begin{array}{c} L_k = 2, 3, 6, 12 \quad r = 4. \end{array}
\] (16)

(a) Model specifications

<table>
<thead>
<tr>
<th>Lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>9</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ): Term</td>
<td>-0.0698</td>
<td>-0.0523</td>
<td>-0.0174</td>
<td>-0.0287</td>
<td>-0.0672</td>
<td>-0.0315</td>
<td>-0.0576</td>
<td>-0.0014</td>
<td>0.9989</td>
<td></td>
</tr>
<tr>
<td>Lag</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>( \beta ): Term</td>
<td>-0.8655</td>
<td>0.0035</td>
<td>-0.0206</td>
<td>0.0353</td>
<td>-0.0112</td>
<td>0.0061</td>
<td>0.0468</td>
<td>-0.0078</td>
<td>0.0237</td>
<td>0.8911</td>
</tr>
<tr>
<td>Lag</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma ): Term</td>
<td>0.2490</td>
<td>-0.0503</td>
<td>0.2339</td>
<td>-0.7541</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lag</td>
<td>0.0000263</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSQ</td>
<td>0.0807512</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Testing the residuals

Mean \( : 0.000038 \)

Variance \( : 0.000263 \)

\(^1\)See appendix 2.

\(^2\)In this paper, \( \sum (m_t - \hat{m}_t)^2 \) is minimized rather than \( \sum (y_t - \hat{y}_t)^2 \), as is customary. Studies have shown that this improves the quality of the predictions.
Test for normal distribution of the residuals
Chi-square test: 0.5953 < 0.9500

Autocorrelation coefficients

<table>
<thead>
<tr>
<th>Llag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lag</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.0376</td>
<td>-0.0035</td>
<td>0.0773</td>
<td>-0.0695</td>
<td>0.0167</td>
<td>0.0337</td>
<td>0.0084</td>
<td>-0.0087</td>
<td>0.0212</td>
<td>-0.0394</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Std. deviation (lower limit) of the ac coefficients: 0.0595

Approximate test whether model equation is suitable
Chi-square test: 0.1089 < 0.9500

All the tests carried out on the residuals corroborate this model equation.

(c) Percentage deviations of the predictions

Mean : 0.0016
Std. deviation : 0.8706
Max. neg. value : -2.5092
Max. pos. value : 2.7147

Only 4 of the 300 predictions show a deviation greater than ±2.5% and 90% of all the predictions show a deviation smaller than ±1.5%. Eq. (16) seems to be an excellent equation for controlling the money supply. However, if longer term predictions (i.e., 2 to 6 months ahead) are attempted, the equation proves unstable. Studies have shown that this instability stems from the moving average parameters lag = 6 and 12.

\[ L_i = 1, 6, 12, \quad p = 3 \]
\[ L_j = 0, 3, 6, 9, 12, \quad q = 5 \]

(a) Model specification

<table>
<thead>
<tr>
<th>Llag : 1</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>-0.0728</td>
<td>0.1446</td>
</tr>
<tr>
<td>Output</td>
<td>Regressive terms x</td>
<td></td>
</tr>
</tbody>
</table>
H.-J. Büttler et al., A money multiplier model

<table>
<thead>
<tr>
<th>Lag</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_j$: Term</td>
<td>-0.8279</td>
<td>0.0846</td>
<td>0.1300</td>
<td>0.0557</td>
<td>0.6148</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.0000755</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SSQ$</td>
<td>0.1123587</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Testing the residuals

Mean : 0.000112

Variance : 0.000365

Test for normal distribution of the residuals

Chi-square test: 0.4671 <0.9500

Autocorrelation coefficients

<table>
<thead>
<tr>
<th>Lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC:</td>
<td>-0.0395</td>
<td>0.0838</td>
<td>-0.0050</td>
<td>-0.0035</td>
<td>0.0022</td>
<td>0.0186</td>
<td>0.0141</td>
<td>0.0286</td>
<td>0.0369</td>
<td>-0.0603</td>
</tr>
<tr>
<td>AC:</td>
<td>0.0531</td>
<td>-0.1947</td>
<td>0.0605</td>
<td>-0.0991</td>
<td>-0.0110</td>
<td>0.0481</td>
<td>0.0003</td>
<td>0.0289</td>
<td>0.0080</td>
<td>-0.0913</td>
</tr>
</tbody>
</table>

Std. deviation (lower limit) of the AC coefficients: 0.0579

Approxiimate test whether model equation is suitable

Chi-square test: 0.8969 <0.9500

In spite of a marked autocorrelation in the case of lag = 12, this equation passes all the tests.

(c) Percentage deviations of the predictions

Mean : 0.0015

Std. deviation : 1.0050

Max. neg. value : -2.8573

Max. pos. value : 3.4294

Although the results are not as good as those obtained with eq. (16), this model comes closer to satisfying the requirements for controlling the money stock. In particular, the longer term predictions are stable.

(d) 'Ex-ante' predictions

In order to test the forecasting power of eq. (17) the parameters are estimated using the data for periods 1–300. 'Ex-ante' predictions are then
made for the periods 301–320, assuming that the adjusted monetary base is known. The results are presented in table 1 of section 2.

**Parameters estimated using the values for periods 14–300**

<table>
<thead>
<tr>
<th>Lag</th>
<th></th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_j$: Term</td>
<td>-0.0831</td>
<td>0.1437</td>
<td>0.6208</td>
</tr>
<tr>
<td>Output: Regressive terms $x$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lag</td>
<td>0</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>$\beta_j$: Term</td>
<td>-0.8251</td>
<td>0.0826</td>
<td>0.1329</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.0001716</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSQ</td>
<td>0.1039819</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ L_1 = 6, 12, \quad p = 2, \]
\[ L_1 = 0, 1, 3, 6, 9, 12, \quad q = 6, \]
\[ L_2 = 1, 2, \quad r = 2. \quad (18) \]

This equation was estimated for the periods 1950–59, 1960–69, 1970–76 and 1950–1976. The results are presented in table 2. Since no standard deviations are calculated for the estimated parameters, it cannot be tested whether the various estimations differ significantly. However, the parameters appear to be quite stable. ($S^2_e$ = estimation for the variance of the residuals.)

**Table 2**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$ = 6</td>
<td>0.1419</td>
<td>0.0794</td>
<td>0.1369</td>
<td>0.1275</td>
</tr>
<tr>
<td>12</td>
<td>0.6656</td>
<td>0.5595</td>
<td>0.6407</td>
<td>0.7846</td>
</tr>
<tr>
<td>$L_2$ = 0</td>
<td>-0.8359</td>
<td>-0.8646</td>
<td>-0.8531</td>
<td>-0.8416</td>
</tr>
<tr>
<td>1</td>
<td>0.0596</td>
<td>-0.0450</td>
<td>0.0347</td>
<td>0.0834</td>
</tr>
<tr>
<td>3</td>
<td>0.0783</td>
<td>0.1637</td>
<td>0.0333</td>
<td>0.1114</td>
</tr>
<tr>
<td>6</td>
<td>0.1290</td>
<td>0.1271</td>
<td>0.0968</td>
<td>0.1339</td>
</tr>
<tr>
<td>9</td>
<td>0.0316</td>
<td>0.2466</td>
<td>0.0270</td>
<td>0.0709</td>
</tr>
<tr>
<td>12</td>
<td>0.6308</td>
<td>0.5748</td>
<td>0.6358</td>
<td>0.6943</td>
</tr>
<tr>
<td>$L_3$ = 1</td>
<td>-0.1076</td>
<td>-0.3299</td>
<td>-0.0707</td>
<td>-0.0093</td>
</tr>
<tr>
<td>2</td>
<td>0.1147</td>
<td>-0.0086</td>
<td>0.1939</td>
<td>0.2999</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-4.23 $\cdot$ 10^{-4}</td>
<td>-6.47 $\cdot$ 10^{-4}</td>
<td>4.48 $\cdot$ 10^{-4}</td>
<td>-7.56 $\cdot$ 10^{-4}</td>
</tr>
<tr>
<td>$S^2_e$</td>
<td>3.62 $\cdot$ 10^{-4}</td>
<td>2.27 $\cdot$ 10^{-4}</td>
<td>3.29 $\cdot$ 10^{-4}</td>
<td>4.66 $\cdot$ 10^{-4}</td>
</tr>
</tbody>
</table>
Appendix 2

Autocorrelation function $AC_y$

<table>
<thead>
<tr>
<th>Lag</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>1.000</td>
<td>0.980</td>
<td>-0.272</td>
<td>-0.167</td>
<td>-0.094</td>
<td>0.089</td>
<td>0.221</td>
</tr>
<tr>
<td>Lag</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>AC</td>
<td>0.028</td>
<td>-0.169</td>
<td>-0.126</td>
<td>-0.028</td>
<td>0.182</td>
<td>0.049</td>
<td>-0.191</td>
</tr>
<tr>
<td>Lag</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>AC</td>
<td>-0.084</td>
<td>-0.016</td>
<td>0.026</td>
<td>0.072</td>
<td>0.181</td>
<td>-0.017</td>
<td>-0.166</td>
</tr>
</tbody>
</table>

Cross-correlation function $CC_{xy}$

<table>
<thead>
<tr>
<th>Lag</th>
<th>-20</th>
<th>-19</th>
<th>-18</th>
<th>-17</th>
<th>-16</th>
<th>-15</th>
<th>-14</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>0.165</td>
<td>0.010</td>
<td>-0.070</td>
<td>-0.032</td>
<td>-0.027</td>
<td>0.039</td>
<td>0.068</td>
</tr>
<tr>
<td>Lag</td>
<td>-13</td>
<td>-12</td>
<td>-11</td>
<td>-10</td>
<td>-9</td>
<td>-8</td>
<td>-7</td>
</tr>
<tr>
<td>CC</td>
<td>0.134</td>
<td>0.124</td>
<td>-0.121</td>
<td>-0.084</td>
<td>-0.041</td>
<td>0.137</td>
<td>-0.119</td>
</tr>
<tr>
<td>Lag</td>
<td>-6</td>
<td>-5</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>CC</td>
<td>-0.128</td>
<td>-0.066</td>
<td>0.064</td>
<td>0.070</td>
<td>0.207</td>
<td>0.223</td>
<td>-0.157</td>
</tr>
<tr>
<td>Lag</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>CC</td>
<td>-0.109</td>
<td>0.248</td>
<td>0.127</td>
<td>0.011</td>
<td>-0.108</td>
<td>-0.151</td>
<td>-0.024</td>
</tr>
<tr>
<td>Lag</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>CC</td>
<td>0.142</td>
<td>0.090</td>
<td>-0.043</td>
<td>-0.233</td>
<td>0.070</td>
<td>0.242</td>
<td>0.016</td>
</tr>
<tr>
<td>Lag</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>CC</td>
<td>-0.042</td>
<td>-0.075</td>
<td>-0.076</td>
<td>-0.077</td>
<td>0.041</td>
<td>0.124</td>
<td></td>
</tr>
</tbody>
</table>

Appendix 3

The tests applied to the residuals serve to check the 'white noise' hypothesis.

Test no. 1

This tests the hypothesis that the residuals stem from a normally distributed population. The method used is the chi-square test.

Test no. 2

This test is carried out to check whether the residuals are autocorrelated. By estimating the standard deviation of the autocorrelation coefficients ($S_{AC} \approx 1/\sqrt{n}$, $n =$ number of observations; Bartlett formula) it is checked whether zero lies within the 95% confidence interval $[AC(K) \pm 1.96(1/\sqrt{n})]$. If this proves to be true for no more than one or two of the thirty $AC$ values printed out, the hypothesis that these $AC$ values are uncorrelated is accepted.

Test no. 3

This tests whether the model equation is suitable. The probability of $Q = \sum_{k=1}^{10} AC(k)^2$, is determined on the basis of the corresponding chi-square distribution. If the probability established in this way is less than 0.95, the model may be regarded as adequate.

Test no. 4

For the models of type $p \geq 0$, $q > 0$, the cross-correlation function $CC_{xy}(t)$
was also determined and it was checked whether or not the terms differ significantly from zero.

References


Revision der Geldmengenstatistik, 1975, Supplement to the Monatsbericht der Schweizerischen Nationalbank, no. 8, Aug.

