Real Effective Exchange Rates of Imports, Exports, and Trade Balance

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Introduction

Since the mid-seventies, international and national authorities as well as private institutions publish regularily indices of "real" foreign-currency prices of the domestic currency, most widely denoted as the real effective exchange rate. In the case of Switzerland, the real effective exchange rate is computed as the trade-weighted arithmetic mean of fifteen "real" exchange rates, the weights being the shares of exports to these countries in total Swiss exports. "Real" exchange rates are computed as foreign-currency prices of the Swiss franc, multiplied by the ratio of domestic of foreign consumer prices.

Computation of such a currency index raises two questions. First, how should the exchange-rate index look like? Since there is no general solution to the determination of the "ideal" price index (Samuelson, 1947), one cannot expect to find a general solution to the determination of the "ideal" real effective exchange rate, which is an index of relative prices in terms of the domestic currency. The problem is then to find a functional form of the exchange-rate index which satisfies some statistical criteria (or axioms). This is the "statistical problem" of the real effective exchange rate dealt with, for instance, in Vartia and Vartia (1984). Secondly, what kind of weights should be used in the currency basket? This is the "economic problem" of the real effective exchange rate to be discussed in this paper. Obviously, the choice of weights depends on the intended use of the real effective exchange rate (cf. Rhomberg, 1976, Maciejewski, 1983). For instance, Niehans (1983) has shown that the real effective exchange rate relating to changes in the trade balance not only depends on exports but on imports and price elasticities as well. It is not clear, however, whether a rise in that real effective exchange rate would bring about an improvement of the trade balance.

The purpose of this paper is to construct several real effective exchange rates which relate, one-to-one, to imports, exports, and trade balance, respectively. They are derived from a system of countries in which each country's demand for imports and domestic goods is determined by the expenditure system of a representative · household. With the help of the aggregation conditions of the household's

expenditure system, the index weights are defined in such a way that the elasticity of real imports and exports with respect to the corresponding real effective exchange rate is, in absolute value, equal to one. Some index weights can, in general, be negative, but all weights are positive if all tradeable goods are gross substitutes and non-inferior. Moreover, the index weights depend on price elasticities, trade shares, and other characteristics not yet considered.

The model is outlined in the next section. Section 3 derives the "ideal" deflator of aggregate nominal imports, the real effective exchange rate of aggregate real imports, and a condition for an abnormal terms-of-trade effect on real imports. In section 4, two expressions for the real effective exchange rate of aggregate real exports and a condition for an abnormal terms-of-trade effect on real exports are shown. In the last section, the real effective exchange rate of the trade balance, a reconsideration of the Marshall/Lerner condition, and some figures of trade characteristics used in the text are given.

Model

There are n countries each producing a homogeneous and internationally tradeable good $X_i (i := 1 \ldots n)$ whose price in terms of local currency is $p_i (i := 1 \ldots n)$. The exchange rate $e_{ij} (i, j := 1 \ldots n, i \neq j)$ is defined as the price of one unit of the jth currency in terms of the ith currency. For instance, if index i refers to Switzerland and index j to Germany, then e_{ij} denotes the Swiss-franc price of the German mark and e_{ji} the German-mark price of the Swiss franc. To simplify the notation, the own-currency price of the own currency, defined to be one, is introduced:

$$e_{ii} := 1, \ i := 1 \ldots n. \tag{1}$$

By the triangular arbitrage condition, any exchange rate can be written in terms of a reference currency, the kth currency, say:

$$e_{ij} = \frac{e_{kj}}{e_{ki}}, \ i, j, k := 1 \ldots n. \tag{2}$$

For instance, if indices i, j, and k refer to Switzerland, Germany, and the United States, respectively, then equ. (2) says that the Swiss-franc price of the German mark is equal to the ratio of the dollar price of the German mark to the dollar price of the Swiss franc.

Households of each country are assumed to consume both domestic and foreign goods. Neglecting transportation cost and import tariffs, the price of the imported

good from the jth country in terms of the ith currency (p_{ij}) is equal to the price of the imported good in terms of its own currency multiplied by the corresponding exchange rate:

$$p_{ij} := e_{ij} p_j, \quad i, j := 1 \ldots n. \tag{3}$$

The ith country's import of good X_j from the jth country is denoted as $x_{ji}(i, j := 1 \ldots n, i \neq j)$ and its demand for the domestic good as $x_{ii}(i := 1 \ldots n)$. A representative household of the ith country is assumed to maximize its utility u_i with regard to all tradeables subject to the budget constraint:

$$\max u_i(x_{ji}, j := 1 \ldots n) \quad \text{s.t.} \quad y_i^n \geq \sum_{j=1}^{n} p_{ij} x_{ji}, \tag{4}$$

where y_i^n is nominal income (expenditure) in terms of the ith currency. Imports and domestic spending on the domestic good determine the expenditure system of the ith country's representative household, the aggregation condition of which will be exploited in the sequel. Using the price of the domestic good $p_{ii} := e_{ii} p_i = p_i$ as the unit of account, the ith country's demand for imported and domestic goods can be written as

$$x_{ji} = x_{ji} \left[\frac{y_i^n}{p_{ii}}, \frac{p_{ik}}{p_{ii}} (k := 1 \ldots n, k \neq i) \right]$$

$$= x_{ji}[y_i, r_{ik}(k := 1 \ldots n, k \neq i)], \quad j := 1 \ldots n, \tag{5}$$

where $y_i := y_i^n / p_i$ is "real" income (expenditure) and $r_{ik} := p_{ik}/p_{ii} = e_{ik} p_k / p_i$ are the ith country's bilateral terms of trade. In the absence of transportation cost and import tariffs, the triangular arbitrage condition holds for any bilateral terms of trade as well:

$$r_{ij} = \frac{r_{kj}}{r_{ki}}, \quad i, j, k := 1 \ldots n. \tag{6}$$

In the sequel, the effect of the terms of trade on aggregate imports, aggregate exports, and trade balance is studied on the assumption that "real" income is held

constant[1]. Changes in the terms of trade can be caused by either changes in exchange rates, defining the nominal effective exchange rate, or changes in both prices and exchange rates, determining the real effective exchange rate.

It should be noted that the results to be derived below do not change qualitatively if non-tradeable goods and imported goods serving as inputs in domestic production are introduced. In the case of imported intermediate goods, the formula for the real effective exchange rate of exports turns out to be simpler than that in the case of final demand.

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[1] In order to eliminate the income effect in empirical applications, it would be convenient to use the utility-compensated demand function, i.e., the Hicksian demand curve, provided that one is willing to accept a certain condition to be explained below. For convenience, the country index will be deleted here. The Hicksian demand $\hat{x}_j$ is derived from minimizing expenditure A subject to the restraint that the utility attained be at least as great as a predetermined value $\bar{u}$:

$$\min A = \sum_j p_j x_j \quad \text{s.t.} \quad u(x_j) \geqq \bar{u}. \tag{i}$$

The minimization yields Hicksian demand functions for imports $\hat{x}_j$ $(j := 1 \ldots n, j \neq i)$ and domestic good $\hat{x}_i$, which are functions of utility level and prices. If income y^n and expenditure A are the same, then both expenditure minimization of equation (i) and utility maximization of equation (4) yield the same values for goods demanded and utility attained. Using the first-order conditions for utility maximization of (4) and the change in expenditure, the relative change in the Hicksian demand function becomes:

$$d(\ln \hat{x}_j) = \frac{\lambda y^n}{\bar{u}} E(\bar{x}_j, \bar{u}) \, [d(\ln y^n) - \sum_k \delta_k d(\ln p_k)] + \sum_k E(\hat{x}_j, p_k) d(\ln p_k), \quad j := 1 \ldots n, \tag{ii}$$

where δ_k is the value share of the kth commodity in total expenditure and $E(x, y)$ is the elasticity of x with regard to y. The term in square brackets is the change in utility-compensated or real income. On the assumption that both value shares and

$$\lambda y^n E(\hat{x}_j, \bar{u}) / \bar{u}$$

are constant values, the income deflator p_y can be written as the weighted geometric mean of prices, the weights being value shares:

$$p_y = \prod_k (p_k)^{\delta_k}.$$

This is the formula most widely used in both theoretical and empirical work. In view of the above assumption and Samuelson's (1947) demonstration that there exists no general solution to the determination of the ideal income deflator, the empirical application of the utility-compensated demand function is rather limited.

Imports

Total nominal imports of the ith country, the home country say, in terms of domestic currency is equal to the sum of bilateral imports:

$$m_i^n = \sum_{j \neq i} p_{ij} x_{ji}. \tag{7}$$

To simplify the notation, summations and multiplications which run always from 1 to n, i.e.,

$$\sum_{\substack{j=1 \\ j \neq 1}}^{n} \text{ and } \prod_{\substack{j=1 \\ j \neq 1}}^{n} \text{ are written as } \sum_{j \neq i} \text{ and } \prod_{j \neq i}, \text{ respectively.}$$

Defining the home country's import shares as

$$\alpha_{ji} := \frac{p_{ij} x_{ji}}{m_i^n}, \quad \sum_{j \neq i} \alpha_{ji} = 1, \tag{8}$$

then the relative change of total nominal imports is given by:

$$d(\ln m_i^n) = \sum_{j \neq i} \alpha_{ji} d(\ln p_{ij}) + \sum_{j \neq i} \alpha_{ji} d(\ln x_{ji})$$

$$:= d(\ln p_{i.}) + d(\ln m_i). \tag{9}$$

The first term on the right-hand side denotes the relative price change of total nominal imports and the second term the relative change in total real imports. Using the relationship $d(\ln r_{ij}) = d(\ln p_{ij}) - d(\ln p_{ii})$, the deflator of aggregate nominal imports becomes:

$$d(\ln p_{i.}) = d(\ln p_i) + \sum_{j \neq i} \alpha_{ji} d(\ln r_{ij})$$

$$:= d(\ln p_i) + d(\ln r_{i.}), \tag{10a}$$

and for constant import shares:

$$p_{i.} = p_i r_{i.}, \quad r_{i.} = \prod_{j \neq i} (r_{ij})^{\alpha_{ji}}. \tag{10b}$$

The *first result* is obtained: If import shares are constant, then the real effective exchange rate $r_{i.}$, multiplied by the price of the domestic good p_i, is the ideal deflator of aggregate nominal imports[2].

Next, the relative change in total real imports is considered. Holding "real" income constant, differentiation of equ. (5) yields:

$$d(\ln m_i) = \sum_{j \neq i} \alpha_{ji} \sum_{k \neq i} E(x_{ji}, r_{ik}) d(\ln r_{ik})$$

$$= \sum_{k \neq i} \left[\sum_{j \neq i} \alpha_{ji} E(x_{ji}, r_{ik}) \right] d(\ln r_{ik}), \tag{11}$$

where $E(x, r)$ is the elasticity of x with regard to r, i.e., $E(x, r) := d(\ln x)/d(\ln r)$. Since we are dealing with an expenditure system, the Cournot aggregation is applied, written as:

$$-p_{ik} x_{ki} = \sum_{j \neq i} p_{ij} x_{ji} E(x_{ji}, p_{ik}) + p_{ii} x_{ii} E(x_{ii}, p_{ik}), \quad k := 1 \ldots n, \ k \neq i. \tag{12}$$

Let β_{ii} be the ratio of domestic spending on the domestic good $(p_{ii} x_{ii})$ to aggregate nominal imports (m_i^n) of the *ith* country

$$\beta_{ii} := p_{ii} x_{ii} / m_i^n = (y_i^n / m_i^n) - 1 > 0, \tag{13}$$

then equ. (12) can be written in terms of imports shares and β_{ii}, when dividing (12) through aggregate nominal imports. Henceforth, β_{ii} will be denoted as the domestic-demand-to-imports ratio. Since the price of the domestic good (p_{ii}) is not included in the elasticities in equ. (12), the elasticities with regard to prices are equal to those with regard to bilateral terms of trade:

$$E(x, p) = E(x, r). \tag{14}$$

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[2] The geometric rather than the arithmetic mean has the advantage of being valid in both absolute and relative values. Let $r_{ij}(I)$ be the index and $r_{ij}(t_0)$ the absolute value at time t_0 (the base period), then the real effective exchange rate can be written

$$r_{i.} = \prod_{j \neq i} (r_{ij})^{\alpha_{ji}} = \prod_{j \neq i} [r_{ij}(I) r_{ij}(t_0)]^{\alpha_{ji}}$$

$$= \prod_{j \neq i} [r_{ij}(I)]^{\alpha_{ji}} \prod_{j \neq i} [r_{ij}(t_0)]^{\alpha_{ji}} = \left[\prod_{j \neq i} r_{ij}(I)^{\alpha_{ji}} \right] r_{i.}(t_0).$$

Hence, the index of the real effective exchange rate

$$r_{i.}(I) := r_{i.} / r_{i.}(t_0) = \prod_{j \neq i} [r_{ij}(I)]^{\alpha_{ji}}$$

has the same form as the real effective exchange rate itself.

Substituting equations (13) and (14) into (12), (12) into (11), the real effective exchange rate of imports can be defined:

$$d(\ln m_i) = - \sum_{k \neq i} [\alpha_{ki} + \beta_{ii} E(x_{ii}, r_{ik})] d(\ln r_{ik})$$

$$:= - \sum_{k \neq i} g_{ki} d(\ln r_{ik})$$

$$:= - d(\ln r_{im}). \tag{15a}$$

The term in square brackets on the first line in equ. (15a) is identified as the index weight (g_{ki}) of the real effective exchange rate of imports (r_{im}). By definition, the elasticity of aggregate real imports with regard to the real effective exchange rate is equal to minus one, irrespective of whether the terms-of-trade effect on aggregate real imports is normal or not. Since import shares and the domestic-demand-to-imports ratio are positive, the sign of the index weight depends on both sign and value of the elasticity of domestic spending on the domestic good with regard to bilateral terms of trade.

Hence, the *second result* is obtained: The real effective exchange rate of imports r_{im} depends on import shares α_{ki}, the domestic-demand-to-imports ratio β_{ii}, and the elasticities of domestic spending on the domestic good with regard to all bilateral terms of trade $E(x_{ii}, r_{ik})$. If domestic and foreign goods are gross substitutes, i.e., $E(x_{ii}, r_{ik}) > 0$, then all index weights g_{ki} are positive and the terms-of-trade effect on total real imports is normal. The elasticity of real imports with regard to the real effective exchange rate $E(m_i, r_{im})$ is defined to be minus one.

If all terms-of-trade elasticities of domestic spending on the domestic good are nil[3], then the real effective exchange rate can be written as the weighted geometric mean of all bilateral terms of trade, provided that import shares be constant values:

$$r_{im} = \prod_{k \neq i} (r_{ik})^{\alpha_{ki}}. \tag{15b}$$

Equation (15b) is the formula most widely used, including import shares as positive index weights only. However, it is likely that neither all cross-price effects are zero nor that the domestic good is a gross substitute for all foreign goods. In the latter case, some index weights might be negative.

For these reasons, the index weights should periodically be derived from regressing aggregate real imports on all bilateral terms of trade of the home country, including domestic income and other possible explanatory variables. For instance, assuming constant values for α_{ki}, β_{ii}, and $E(x_{ii}, r_{ik})$ during the observation period in

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[3] Given a Cobb-Douglas utility function, the home country's demand functions for foreign and domestic commodities depend on their own prices only, i.e., $E(x_{ii}, r_{ik}) = 0$ for $k \neq i$.

question, a log-log regression immediately yields the index weigths, from which the real effective exchange rate in "geometric form"

$$r_{im} = \prod_{k \neq i} (r_{ik})^{g_{ki}}$$

(15c)

is obtained. Consequently, some index weights in (15c) could be negative. Moreover, the sum of weights is, in general, not equal to one.

It is possible to derive a simple condition for an abnormal terms-of-trade effect on aggregate real imports if all bilateral terms of trade rise exactly to the same extent. For the sake of convenience, let the relative change in the bilateral terms of trade be equal to one, i.e., $d(\ln r_{ik}) = 1$, $k := 1 \dots n$, $k \neq i$. Since the import shares sum up to one, equ. (15a) now becomes:

$$d(\ln m_i) = -[1 + \beta_{ii} \sum_{k \neq i} E(x_{ii}, r_{ik})].$$

(16a)

Using the Euler aggregation (homogeneity)

$$E(x_{ii}, p_{ii}) = -[E(x_{ii}, y_i) + \sum_{k \neq i} E(x_{ii}, r_{ik})]$$

(17)

and the Slutsky equation

$$E(x_{ii}, p_{ii}) = \bar{E}(x_{ii}, p_{ii}) - \delta_{ii} E(x_{ii}, y_i),$$

(18)

where $\bar{E}(.,.)$ denotes the utility-compensated elasticity, and introducing the value share of the domestic good in total expenditure of the home country:

$$\delta_{ii} := p_{ii} x_{ii} / y_i^n,$$

(19)

then equation (16a) is written as (16b):

$$d(\ln m_i) = -1 + \beta_{ii} [\bar{E}(x_{ii}, p_{ii}) + (1 - \delta_{ii}) E(x_{ii}, y_i)].$$

(16b)

Since the utility-compensated own-price effect is negative, i.e., $\bar{E}(x_{ii}, p_{ii}) < 0$, aggregate real imports are a normal "good", provided that the domestic good is non-superior. As a consequence, the weighted imports being the mirror-image of domestic spending on the domestic good must be non-inferior (cf. equation 16d). Suppose that aggregate real imports react abnormally, that is, increase as all import prices go up by the same relative amount, then a simple condition is obtained from (16b):

$$E(x_{ii}, y_i) > \frac{1 + \beta_{ii}}{\beta_{ii}} [1 - \beta_{ii} \bar{E}(x_{ii}, p_{ii})] \geq \frac{1 + \beta_{ii}}{\beta_{ii}} = \frac{1}{\delta_{ii}} > 0.$$

(16c)

In order to obtain an abnormal terms-of-trade effect on aggregate real imports, the income elasticity of domestic spending on the domestic good must exceed the reciprocal of the value share of the domestic good. Given a plausible value for the

Swiss domestic-demand-to-imports ratio of one (cf. Table 1), the following income elasticities are calculated from (16c) for various utility-compensated own-price elasticities:

$\bar{E}(x_{ii}, p_{ii}): 0, -0.5, -1, -1.5,$

$E(x_{ii}, y_i) > 2, 3, 4, 5.$

Even if the utility-compensated own-price elasticity of domestic spending on the domestic good is zero, then the income elasticity of domestic demand has to be greater than two, which seems to be implausible a value for aggregate consumption. Finally, using the Engel aggregation, equation (16d) is obtained

$$\sum_{k \neq i} \alpha_{ki} E(x_{ki}, y_i) \leq (1 + \beta_{ii}) \beta_{ii} \bar{E}(x_{ii}, p_{ii}) < 0, \qquad (16d)$$

which says that the weighted sum of income elasticities of the home country's imports must be negative, the weights being the import shares. Hence, aggregate real imports must be an inferior "good" in order to react abnormally to changes in the terms of trade.

Exports

Total real exports of the *ith* country are:

$$x_i = \sum_{j \neq i} x_{ij}. \qquad (20)$$

Using the definition of the home country's export shares

$$\xi_{ij} := \frac{p_i x_{ij}}{p_i x_i} = \frac{p_i x_{ij}}{x_i^n}, \quad \sum_{j \neq i} \xi_{ij} = 1 \qquad (21)$$

the relative change in aggregate real exports is:

$$d(\ln x_i) = \sum_{j \neq i} \xi_{ij} d(\ln x_{ij}). \qquad (22)$$

Provided that "real" income is held constant, differentiation of equation (5) yields:

$$d(\ln x_i) = \sum_{j \neq i} \xi_{ij} \sum_{k \neq j} E(x_{ij}, r_{jk}) d(\ln r_{jk}). \qquad (23)$$

Equation (23) includes bilateral terms of trade of foreign countries. Using the relation $d(\ln r_{jk}) = d(\ln r_{ik}) - d(\ln r_{ij})$ given in equation (6), the relative change in

exports can be written as a function of all bilateral terms of trade of the home country:

$$d(\ln x_i) = \sum_{j \neq i} \left[\sum_{k \neq i, j} \xi_{ik} E(x_{ik}, r_{kj}) - \xi_{ij} \sum_{k \neq j} E(x_{ij}, r_{jk}) \right] d(\ln r_{ij})$$

$$:= \sum_{j \neq i} h_{ij} d(\ln r_{ij})$$

$$:= d(\ln r_{ix}) \tag{24a}$$

The expression in square brackets on the first line in equation (24a) is defined as the index weight h_{ij}, consisting of two terms. In the first term, there are cross-price effects only, excluding the bilateral terms of trade of the home country. If the exported domestic good is a gross substitute for all tradeables, then the first term is positive. In the second term, there are own-price effects as well. The Euler aggregation for x_{ij} is:

$$E(x_{ij}, p_{jj}) = -[E(x_{ij}, y_j) + \sum_{k \neq j} E(x_{ij}, r_{jk})], \quad j := 1 \ldots n, j \neq i. \tag{25}$$

If the exported domestic good is a gross substitute for all traded foreign goods, i.e., $E(x_{ij}, p_{jj}) > 0$, and if the domestic good is non-inferior in foreign countries' imports, then the second term is positive.

Hence, the *third result* is obtained: The real effective exchange rate of aggregate real exports r_{ix} depends on both exports shares ξ_{ik} and terms-of-trade elasticities of foreign imports $E(x_{ik}, r_{kj})$. If the exported domestic good is a gross substitute for all foreign goods and non-inferior, then all index weights h_{ij} are positive and the terms-of-trade effect on total real exports is normal. The elasticity of aggregate real exports with regard to the real effective exchange rate $E(x_i, r_{ix})$ is defined to be one.

Suppose that (i) all cross-price effects were zero, (ii) all own-price elasticities of foreign imports were equal to minus one, and (iii) export shares were constant values, then the real effective exchange rate of exports can be written as the weighted geometric mean of all bilateral terms of trade of the home country, the weights being export shares:

$$r_{ix} = \prod_{j \neq i} (r_{ij})^{\xi_{ij}}. \tag{24b}$$

This is the formula most widely used, including positive index weights only. However, if the above mentioned assumptions do not hold, negative index weights are more likely in the case of exports than imports.

Again, index weights should periodically be derived from regressing aggregate real exports on all bilateral terms of trade of the home country, including aggregate foreign income and other possible explanatory variables. The aggregation of foreign income can be done in the same way as for the real effective exchange rate. If

one assumes constant values for all trade characteristics in equation (24a), then a log-log regression yields the index weights h_{ij}, from which the real effective exchange rate in "geometric form"

$$r_{ix} = \prod_{j \neq i} (r_{ij})^{h_{ij}}$$
(24c)

is obtained. Again, the index weights do not, in general, sum up to unity.

There is another useful expression for the real effective exchange rate of exports when exploiting the symmetry of utility-compensated price effects, i.e.,

$$\frac{\partial x_{ij}}{\partial p_{jk}} + \frac{\partial x_{ij}}{\partial y_j^n} x_{kj} = \frac{\partial x_{kj}}{\partial p_{ji}} + \frac{\partial x_{kj}}{\partial y_j^n} x_{ij}, \quad k := 1 \ldots n.$$
(26a)

Let δ_{ij} be the value share of the ith commodity in total expenditure of the jth country, i.e.,

$$\delta_{ij} := \frac{x_{ij} p_{ji}}{y_j^n}$$
(26b)

Using the import shares given in equation (8) and the value shares given in (26b), the symmetry condition becomes equation (26c):

$$E(x_{ij}, r_{jk}) = \frac{\alpha_{kj}}{\alpha_{ij}} [E(x_{kj}, r_{ji}) + \delta_{ij}\{E(x_{kj}, y_j) - E(x_{ij}, y_j)\}].$$
(26c)

Substituting (26c) into the second term in (24a), applying both the Cournot and Engel aggregation, the real effective exchange rate of exports can also be written as equation (27):

$$d(\ln x_i) = \sum_{j \neq i} \left[\sum_{k \neq i, j} \xi_{ik} E(x_{ik}, r_{kj}) + \frac{\xi_{ij}}{\alpha_{ij}} \{\beta_{jj} [E(x_{jj}, r_{ji}) + \delta_{ij} E(x_{jj}, y_j)] \right.$$

$$\left. + \delta_{ij} E(x_{ij}, y_j)\} \right] d(\ln r_{ij})$$

$$= \sum_{j \neq i} \left[\sum_{k \neq i, j} \xi_{ik} E(x_{ik}, r_{kj}) + \frac{\xi_{ij}}{\alpha_{ij}} \{\beta_{jj} \bar{E}(x_{jj}, r_{ji}) + \delta_{ij} E(x_{ij}, y_j)\} \right] d(\ln r_{ij})$$

$$:= \sum_{j \neq i} h_{ij} d(\ln r_{ij})$$

$$:= d(\ln r_{ix}).$$
(27)

The index weight is represented by the expression in square brackets. The first term is equal to the first term of equation (24a), whereas the second term is now a function of utility-compensated cross-price elasticities of foreign countries' spend-

ing on their own goods, as well as of income elasticities of foreign imports of the domestic good.

Hence, the *fourth result* is obtained: Apart from the factors considered in the third result, the real effective exchange rate of exports r_{ix} can be written as a function of the share of the domestic good in foreign countries' imports α_{ij}, the value share of the domestic good in foreign countries' total expenditure δ_{ij}, the foreign contries' domestic-demand-to-imports ratio β_{jj}, the terms-of-trade elasticities of foreign countries' spending on their own goods $E(x_{jj}, r_{ji})$, the income elasticities of foreign countries' spending on their own goods $E(x_{jj}, y_j)$, and the income elasticities of foreign countries' imports of the domestic good $E(x_{ij}, y_j)$.

Again, a simple condition for an abnormal terms-of-trade effect on aggregate real exports can be obtained when considering an equal relative change in all bilateral terms of trade of the home country, i.e., $d(\ln r_{ij}) = 1, j := 1 \ldots n, j \neq i$. Using the Euler aggregation, the Slutsky equation and the symmetry condition, the first term of the expression in square brackets in equation (27) can be written as equation (28):

$$\sum_{j \neq i} \sum_{k \neq i, j} \xi_{ik} E(x_{ik}, r_{kj}) =$$

$$- \sum_{k \neq i} \xi_{ik} \left[\frac{\beta_{kk}}{\alpha_{ik}} \bar{E}(x_{kk}, r_{ki}) + \bar{E}(x_{ik}, r_{ki}) + (1 - \delta_{ik} - \delta_{kk}) E(x_{ik}, y_k) \right]. \tag{28}$$

Substituting equation (28) into (27) and using the relation $\delta_{ij} = (1 - \delta_{jj})\alpha_{ij}$, the relative change in aggregate real exports becomes the following simple expression:

$$d(\ln x_i) = - \sum_{j \neq i} \xi_{ij} [\bar{E}(x_{ij}, r_{ji}) - \delta_{ij} E(x_{ij}, y_j)]. \tag{29a}$$

Since the utility-compensated own-price elasticities of foreign countries' imports of the domestic good are negative, i.e., $\bar{E}(x_{ij}, r_{ji}) < 0$, the domestic good only needs to be non-inferior in foreign countries' imports, i.e., $E(x_{ij}, y_j) \geqq 0$, in order that total real exports are a normal "good". Considering negative total real exports in (29a), the simple condition reads:

$$\sum_{j \neq i} \xi_{ij} \delta_{ij} E(x_{ij}, y_j) < \sum_{j \neq i} \xi_{ij} \bar{E}(x_{ij}, r_{ji}) \leq 0. \tag{29b}$$

In order to obtain an abnormal terms-of-trade effect on aggregate real exports, the sum of trade-weighted income elasticities of exports (= foreign countries' imports of the domestic good) must be negative, the weights being both export shares and value shares of the domestic good in foreign countries' total expenditure.

Trade Balance

The nominal trade balance of the home country in terms of domestic currency c_i^n is the difference between total nominal exports x_i^n and imports m_i^n:

$$c_i^n = x_i^n - m_i^n = \sum_{j \neq i} p_{ii} x_{ij} - \sum_{j \neq i} p_{ij} x_{ji}. \tag{30}$$

Let total imports be initially a fraction λ_i of total exports, i.e., $m_i^n = \lambda_i x_i^n (\lambda_i > 0)$. Trade was, therefore, balanced if λ_i was equal to one at the outset. The change in the nominal trade balance relative to total nominal exports can be written as:

$$\frac{dc_i^n}{x_i^n} = d(\ln p_i) + d(\ln x_i) - \lambda_i d(\ln m_i^n). \tag{31}$$

The first term is the relative change in the price of the domestic good, the second term is the relative change in total real exports, and the third term is the relative change in total nominal imports multiplied by the imports-to-exports ratio λ_i. Holding the price of the domestic good constant[4], i.e., $d(\ln p_i) = 0$, substituting equation (24a) for $d(\ln x_i)$ and equations (9), (10a) and (15a) for $d(\ln m_i^n)$ into equation (31), the change in the nominal trade balance relative to total nominal exports is:

$$\frac{dc_i^n}{x_i^n} = \frac{dc_i}{x_i} = \sum_{j \neq i} \left[\sum_{k \neq i,j} \xi_{ik} E(x_{ik}, r_{kj}) - \xi_{ij} \sum_{k \neq j} E(x_{ij}, r_{jk}) + \lambda_i \beta_{ii} E(x_{ii}, r_{ij}) \right] d(\ln r_{ij})$$

$$= \sum_{j \neq i} [h_{ij} + \lambda_i \beta_{ii} E(x_{ii}, r_{ij})] d(\ln r_{ij})$$

$$:= \sum_{j \neq i} z_{ij} d(\ln r_{ij})$$

$$:= d(\ln r_{ic}). \tag{32a}$$

It should be noted that equation (32a) also applies to the change in the real trade balance relative to total real exports, imposing no restriction on the variation of the price of the domestic good (cf. footnote 4). The expression in square brackets is defined to be the index weight z_{ij}. In this representation, the real effective exchange rate of the trade balance does neither depend on the home country's import shares

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[4] If the trade balance is equilibrated at the outset, then $d(\ln p_i)$ in equation (31) and the relative change in the domestic good's price included in $d(\ln m_i^n)$ cancel each other out. This also happens for *any* value of λ_i when the change in the real trade balance is considered. Define the real trade balance as

$$c_i := x_i - \sum_{j \neq i} r_{ij} x_{ji},$$

then the change in the real trade balance at a constant domestic good's price relative to nominal exports $(p_i dc_i / x_i^n) = (dc_i / x_i)$ is equal to the right-hand side of equation (32a).

(most widely used in practice) nor on terms-of-trade elasticities of the home country's imports. All the price elasticities refer to either foreign countries' imports of the domestic good or the home country's demand for its own good.

Hence, the *fifth result* is obtained: The real effective exchange rate of the (nominal or real) trade balance r_{ic} depends on the weights of the real effective exchange rate of exports h_{ij}, the imports-to-exports ratio λ_i, the domestic-demand-to-imports ratio β_{ii}, and all bilateral terms-of-trade elasticities of domestic spending on the domestic good $E(x_{ii}, r_{ij})$. If the domestic good is a gross substitute for all tradeables and non-inferior in foreign countries' imports, then all index weights z_{ij} are positive and the terms-of-trade effect on the (nominal or real) trade balance is normal. The elasticity of the trade balance with respect to the real effective exchange rate, defined to be $E(c_i^n, r_{ic}) := (\partial c_i^n / \partial r_{ic})(r_{ic}/x_i^n)$, is equal to one by definition.

A special result of equation (32a) can be derived if (i) the cross-price elasticities are zero, (ii) own-price elasticities are equal to minus one, and (iii) export shares are constant values:

$$r_{ic} = \prod_{j \neq i} (r_{ij})^{\xi_{ij}}. \tag{32b}$$

This is the special form of the real effective exchange rate of exports given in equation (24b).

In the general case, the index weights can be calculated from those obtained for imports and exports, i.e.,

$$z_{ij} = h_{ij} + \lambda_i [g_{ji} - \alpha_{ji}], \quad j := 1 \ldots n, \quad j \neq i, \tag{32c}$$

where h_{ij} and g_{ji} are the weights defined in equations (15a) and (24a).

In the two-country case, equation (32a) can be related to the Marshall/Lerner condition in order to show that the assumption that domestic and foreign goods are gross substitutes is quite strong. Let the first country be the home country whose change in the trade balance is obtained from equation (32a):

$$\frac{dc_1^n}{x_1^n} = \frac{dc_1}{x_1} = [-E(x_{12}, r_{21}) + \lambda_1 \beta_{11} E(x_{11}, r_{12})] d(\ln r_{12}). \tag{33}$$

By (33), a deterioration of the terms of trade improves the trade balance if the term in square brackets is positive. As for the Marshall/Lerner condition, we take for granted in the following discussion that the home country's import and export consist of normal goods. In other words, the own-price elasticities of both countries' import demand are negative, that is, $E(x_{12}, r_{21})$ and $E(x_{21}, r_{12})$ are negative. Therefore by (33), the domestic good being a gross substitute for the foreign good $[E(x_{11}, r_{12}) > 0]$ is sufficient for a normal terms-of-trade effect on the trade balance. Next, the following relation is derived from using the Cournot aggregation:

$$E(x_{11}, r_{12}) = -[E(x_{21}, r_{12}) + 1]/\beta_{11}. \tag{34}$$

Substitute (34) into (33), let the trade balance initially be equilibrated ($\lambda_1 = 1$), then it follows from the resulting equation that a deterioration of the terms of trade improves the trade balance if the sum of the own-price elasticities of the home country's import and export is, in absolute value, greater than one, that is,

$$- [E(x_{12}, r_{21}) + E(x_{21}, r_{12})] > 1, \tag{35}$$

which is the Marshall/Lerner condition. By (34), the domestic good is a gross substitute for the foreign good if the own-price elasticity of the home country's import is, in absolute value, greater than one. The latter condition, however, is already sufficient to fulfill the Marshall/Lerner condition.

Finally, it is to show whether the trade characteristics, being elements of the index weights, can be regarded as constant values over the observation period in question. In Table 1, the figures of some trade characteristics are given for Switzerland, showing relatively little variation.

Table 1

Trade Characteristics. Annual data from 1972–1983 for Switzerland.
The four countries mentioned below are Switzerland's most important trading partners during the observation period.

	Import shares α in %		Export shares ξ in %	
	Minimum	Maximum	Minimum	Maximum
Germany	28	30	14	20
France	11	14	9	9
Italy	9	10	6	8
U.S.A.	6	8	6	9
			Minimum	Maximum
Imports-to-exports ratio λ			1,00	1,24
Domestic-demand-to-imports ratio β			0,71	1,15

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Abstract

Real Effective Exchange Rates of Imports, Exports, and Trade Balance

This paper considers a system of countries in which each country's demand for both imported and domestic goods is determined by the expenditure system of a representative household. Five different real effective exchange rates are derived: an "ideal" deflator of nominal imports, a real effective exchange rate of real imports, two real effective exchange rates of real exports, and a real effective exchange rate of the trade balance. Nominal effective exchange rates derive from the same expressions as well. Moreover, conditions for normal terms-of-trade effects on aggregate real exports and imports are given. Finally, the paper discusses the possibility of estimating the index weights of the various real effective exchange rates.

Zusammenfassung

Effektive Realkurse der Importe, Exporte und Handelsbilanz

Betrachtet wird ein Mehr-Länder-Modell, in dem die Nachfrage eines Landes nach Importen und heimischen Gütern durch die Bedingungen des Ausgabensystems eines Haushalts bestimmt wird. Es ergeben sich fünf Ergebnisse im Rahmen des Modells: ein effektiver Realkurs zur Deflationierung der nominellen Einfuhr, ein effektiver Realkurs der realen Einfuhr, zwei Formen für einen effektiven Realkurs der realen Ausfuhr und einen effektiven Realkurs der Handelsbilanz. Einfache Bedingungen für die anormale Reaktion von realer Einfuhr und Ausfuhr sowie die Möglichkeit zur empirischen Ermittlung der Indexgewichte werden aufgezeigt.

Résumé

Cours des changes réels, effectifs de l'importation, de l'exportation et de la balance commerciale

Il est tenu compte d'un modèle composé de plusieurs pays dans lequel la demande d'un pays en matière de produits importés et de biens nationaux est définie à partir des exigences du système de dépenses d'un foyer. D'après ce modèle, cinq résultats sont obtenus: un cours réel effectif déflationniste des importations nominales, un cours réel effectif des importations réelles, deux formes de cours réel effectif des exportations réelles et un cours réel effectif de la balance commerciale. Sont mises en évidence les conditions simples d'une réaction anormale des importations et des exportations réelles, ainsi que la possibilité d'une recherche empirique des indices de poids.